# LECTURE 01

# 21CSE451T

# FUZZY LOGIC AND ITS APPLICATIONS

## Unit 01:  Introduction

| # | Topic |
|---|---|
| 1.1 | The Case for Imprecision |
| 1.2 | The Utility of Fuzzy Systems |
| 1.3 | Limitations of Fuzzy Systems |
| 1.4 | Uncertainty and Information |
| 1.5 | Fuzzy Sets and Membership |
| 1.6 | Chance versus Fuzziness |

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# SYLLABUS



| | |
|---|---|
| ***Unit-1 - Introduction*** | ***9 Hour*** |
| *The Case for Imprecision, The Utility of Fuzzy Systems, Limitations of Fuzzy Systems, Uncertainty and Information, Fuzzy Sets and Membership, Chance versus Fuzziness - Fuzzy Sets: Fuzzy Set Operations, Properties of Fuzzy Sets, Noninteractive Fuzzy Sets, Alternative Fuzzy Set Operations - Fuzzy Relations: Crisp Relations, Fuzzy Relations, Fuzzy Tolerance and Equivalence Relations, Value Assignments, Problems on fuzzy relation - Membership function – various forms –fuzzification – defuzzification to crisp sets.* | |
| ***Unit-2 - Logic and Fuzzy Systems*** | ***9 Hour*** |
| *classical logic, fuzzy logic, fuzzy systems – Development of Membership functions: membership value assignments, intuition, Inference, rank ordering – Automated Methods for Fuzzy Systems: Definitions, Batch Least Squares Algorithm, Recursive Least Squares Algorithm, Gradient Method, Learning From Example, Modified Learning From Example, Problems on logic and fuzzy systems* | |
| ***Unit-3 - Rule-Base Reduction Methods*** | ***9 Hour*** |
| *: Fuzzy Systems Theory and Rule Reduction, Singular Value Decomposition, Combs Method, SVD and Combs Method Examples, problems on SVD and Combs method for rapid inference - Decision Making with Fuzzy Information: Fuzzy Synthetic Evaluation, Fuzzy Ordering, Nontransitive Ranking, Preference and Consensus, Multiobjective Decision Making, Decision Making under Fuzzy States and Fuzzy Actions, problems on decision making with fuzzy information.* | |
| ***Unit-4 - Fuzzy Classification and Pattern Recognition*** | ***9 Hour*** |
| *Classification by Equivalence Relations, Cluster Analysis, Cluster Validity, c-Means Clustering, Fuzzy c-Means, Classification Metric, Similarity Relations from Clustering - Pattern Recognition: Feature Analysis, Partitions of the Feature Space, Single-Sample Identification, Multifeature Pattern Recognition, problems on fuzzy classification and pattern recognition, Case Study: Hand written character recognition using fuzzy logic.* | |
| ***Unit-5 - Fuzzy Control Systems*** | ***9 Hour*** |
| *Control System Design Problem, Control (Decision) Surface, Assumptions in a Fuzzy Control System Design, Simple Fuzzy Logic Controllers, Examples of Fuzzy Control System Design, Aircraft Landing Control Problem - Fuzzy Optimization - Fuzzy Linear Regression – problems on fuzzy optimization and regression, Case study: Robot Navigation using fuzzy logic.* | |

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**Unit-1 - Introduction**

The Case for Imprecision, The Utility of Fuzzy Systems, Limitations of Fuzzy Systems, Uncertainty and Information, Fuzzy Sets and Membership, Chance versus Fuzziness -

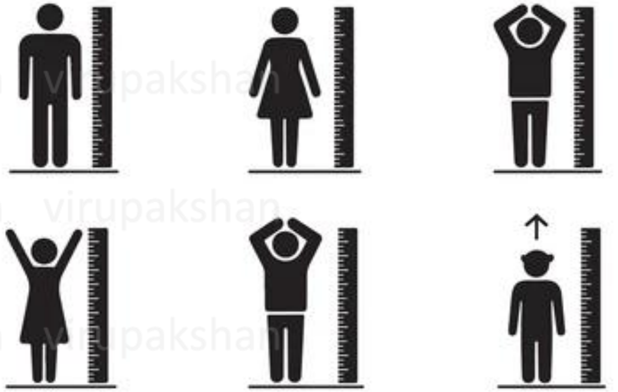
# What are these?


# Have you seen or used a Washing Machine control panel?

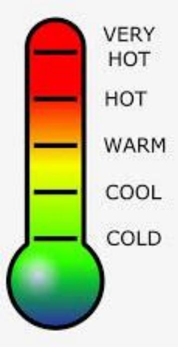
# 1.1.1 The Case for Imprecision

In classical mathematics and computer logic, things are usually treated as **true or false**, **yes or no**, or **0 or 1**.

*But many real-life situations are not so exact.*



Examples:




- A person may be **tall**, **very tall**, or **somewhat tall**.
- A room may be **hot**, **warm**, or **slightly cold**.
- A car may be moving **fast**, **moderately fast**, or **slow**.

These ideas are not precise. They are **imprecise** or **vague**.

Fuzzy logic was introduced to handle such imprecision.

# 1.1.2 The Case for Imprecision

## Classical Logic Example

In crisp logic:

| Temperature | Classification |
| --- | --- |
| 29°C | Not hot |
| 30°C | Hot |
| 31°C | Hot |

Here, the boundary is sharp. At 29°C, the room is not hot, but at 30°C, it suddenly becomes hot. This is not realistic.

## Fuzzy Logic Example

In fuzzy logic:

| Temperature | Degree of Hotness |
| --- | --- |
| 25°C | 0.2 |
| 30°C | 0.6 |
| 35°C | 1.0 |

Here, hotness increases gradually.

# 1.1.3 The Case for Imprecision

**Formula**

A fuzzy **set** $A$ in a universe of discourse $X$ is represented as:

$$A = \{(x, \mu_A(x)) \mid x \in X\}$$

where:

- $x$ is an element.
- $\mu_A(x)$ is the membership value.
- $0 \leq \mu_A(x) \leq 1$ .

**Diagram**

```
Membership
Value
1.0 |                                   ________
    |                                ___/
0.5 |                           ____/
    |                      ____/
0.0 |_____________________/
          20    25    30    35    Temperature
```

This graph shows that temperature gradually becomes hot.

# 1.1.4 The Case for Imprecision

## Worked Example

Suppose the fuzzy set **Hot Temperature** is defined as:

| Temperature | Membership in Hot |
|---|---|
| 20°C | 0.0 |
| 25°C | 0.3 |
| 30°C | 0.7 |
| 35°C | 1.0 |

## Question: How hot is 30°C?

$$\mu_{\text{Hot}}(30) = 0.7$$

So, 30°C is hot to degree 0.7.

python

```python
def hot_temperature(temp):
    if temp <= 20:
        return 0.0
    elif temp >= 35:
        return 1.0
    else:
        return (temp - 20) / (35 - 20)

temperature = 30
membership = hot_temperature(temperature)

print("Temperature:", temperature)
print("Degree of hotness:", membership)
```

# 1.1.5 The Case for Imprecision

**Question:** Explain why fuzzy logic is needed to handle imprecision.

**Answer:**

Fuzzy logic is needed because many real-world concepts do not have sharp boundaries.

For example, temperature cannot always be classified only as hot or not hot. Fuzzy logic allows partial membership, such as 0.6 hot or 0.8 hot, making it useful for modeling human reasoning and real-world uncertainty.

# 1.2.1 The Utility of Fuzzy Systems



A **fuzzy system** is a system that uses fuzzy logic to make decisions based on approximate reasoning.

Fuzzy systems are useful when:

- Exact mathematical models are difficult.
- Human expert knowledge is available.
- Inputs are vague or linguistic.
- Decisions need to be made smoothly.

# 1.2.2 The Utility of Fuzzy Systems



**Applications**

Fuzzy systems are used in:

| Field | Application |
|---|---|
| Control systems | Washing machines, air conditioners |
| Automobiles | Automatic braking systems |
| Medicine | Diagnosis support |
| Robotics | Path planning |
| Decision making | Risk assessment |
| Image processing | Edge detection |
| Finance | Credit scoring |

# 1.2.3 The Utility of Fuzzy Systems

## Basic Structure of a Fuzzy System



Crisp Input

Fuzzification

Fuzzy Rule Base

Inference Engine

Defuzzification

# 1.2.4 The Utility of Fuzzy Systems

Air conditioner control:

Input:

- Room temperature = 32°C

Fuzzy rule:

**IF temperature is hot THEN fan speed is high.**

Output:

- Fan speed = 80%

python

```python
def hot(temp):
    if temp <= 25:
        return 0
    elif temp >= 35:
        return 1
    else:
        return (temp - 25) / 10

temperature = 32
hot_value = hot(temperature)

fan_speed = hot_value * 100

print("Hot membership:", hot_value)
print("Fan speed:", fan_speed)
```

# 1.2.5 The Utility of Fuzzy Systems



**Question:** Draw and explain the basic structure of a fuzzy system.

**Answer:**
A fuzzy system consists of fuzzification, rule base, inference engine, and defuzzification.

Crisp inputs are first converted into fuzzy values.
Fuzzy rules are applied using an inference engine.
Finally, fuzzy output is converted back into a crisp value using defuzzification.

```
Crisp Input
    ↓
Fuzzification
    ↓
Fuzzy Rule Base
    ↓
Inference Engine
    ↓
Defuzzification
    ↓
Crisp Output
```

# 1.3.1 Limitations of Fuzzy Systems



## Main Limitations

### 1. Rule Design Is Difficult

Fuzzy systems depend on rules such as:

```
IF temperature is high THEN fan speed is fast.
```

Creating good rules requires expert knowledge.

### 2. Membership Function Selection Is Subjective

Different designers may choose different membership functions.

Example:

One designer may say: $\mu_{\text{Hot}}(30)=0.6$

Another may say: $\mu_{\text{Hot}}(30)=0.8$

### 3. No Learning by Default

Traditional fuzzy systems do not learn automatically from data unless combined with machine learning.

### 4. Rule Explosion

If there are many input variables, the number of rules increases quickly.

If there are $n$ inputs and each input has $m$ fuzzy sets, then number of rules may be: $m^n$

Example: For 4 inputs and 3 fuzzy sets each: **$3^4=81$** rules may be required.

### 5. Validation Can Be Difficult

It may be difficult to prove that a fuzzy system is always correct.

# 1.3.2 Limitations of Fuzzy Systems

## Diagram: Rule Explosion

```
Inputs increase → Rule combinations increase rapidly

2 inputs, 3 sets each → 3² = 9 rules
3 inputs, 3 sets each → 3³ = 27 rules
4 inputs, 3 sets each → 3⁴ = 81 rules
```

**Worked Example**

Suppose a fuzzy controller has:

- Temperature: low, medium, high
- Humidity: low, medium, high
- Pressure: low, medium, high

Number of variables: $n$=3

Number of fuzzy sets per variable: $m$=3

Number of possible rules: $m^n = 3^3 = 27$

# 1.3.3 Limitations of Fuzzy Systems

python

```
def number_of_rules(inputs, fuzzy_sets_per_input):
    return fuzzy_sets_per_input ** inputs

inputs = 4
sets = 3

rules = number_of_rules(inputs, sets)
print("Number of rules:", rules)
```

# 1.3.4 Limitations of Fuzzy Systems



**Question:** List any four limitations of fuzzy systems.

**Answer:**

Four limitations of fuzzy systems are:

1. rule design requires expert knowledge,
2. membership functions are subjective,
3. rule explosion occurs when many inputs are used, and
4. fuzzy systems do not automatically learn unless combined with learning algorithms.

# 1.4.1 Uncertainty and Information

**Meaning**

Uncertainty means lack of complete knowledge.

In real-world systems, uncertainty can arise due to:

- Incomplete data
- Noisy measurements
- Vague language
- Randomness
- Conflicting information

# 1.4.2 Uncertainty and Information



## Types of Uncertainty

| Type | Meaning | Example |
|---|---|---|
| Random uncertainty | Due to chance | Tossing a coin |
| Fuzzy uncertainty | Due to vagueness | Tall person |
| Incomplete information | Missing data | Sensor failure |
| Ambiguity | Multiple meanings | “High speed” |
| Measurement uncertainty | Error in measurement | Temperature sensor error |

## 1.4.3 Uncertainty and Information

### Information

**Information <u>reduces</u> uncertainty.**

Example:

Before measurement:

`Temperature may be low, medium, or high.`

After measurement:

`Temperature = 32°C.`

The uncertainty is reduced.

# 1.4.4 Uncertainty and Information



**Question:** What is uncertainty? Mention different types of uncertainty.

**Answer:**

Uncertainty is the lack of complete or exact knowledge about a situation.

Types of uncertainty include random uncertainty, fuzzy uncertainty, measurement uncertainty, ambiguity, and incomplete information.

# 1.5.1 Fuzzy Sets and Membership

## Crisp Set

A crisp set has only two membership values:

$$\mu_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$$

Example:

Set of adults:

**Age ≥ 18 → adult**

**Age < 18 → not adult**

## Fuzzy Set

A fuzzy set allows partial membership:

$$0 \le \mu_A(x) \le 1$$

Example:

**Set of young people:**

| Age | Membership in Young |
|---|---|
| 10 | 1.0 |
| 20 | 0.8 |
| 30 | 0.4 |
| 40 | 0.1 |
| 50 | 0.0 |

# 1.5.2 Fuzzy Sets and Membership



## Representation

A fuzzy set can be written as:

$$A = \frac{0.2}{x_1} + \frac{0.5}{x_2} + \frac{1.0}{x_3}$$

This <u>does not mean</u> division.

It means:

- $x_1$ has membership 0.2
- $x_2$ has membership 0.5
- $x_3$ has membership 1.0

## Worked Example

**Let**: $X$={10,20,30,40}

Fuzzy set Young: $$Y = \frac{1.0}{10} + \frac{0.8}{20} + \frac{0.4}{30} + \frac{0.1}{40}$$

**Question**:

What is the membership of 30 in Young?

**Answer**: $\mu_Y(30) = 0.4$

# 1.5. Fuzzy Sets and Membership

## Quick Revision Points

- Crisp sets allow only 0 or 1 membership.
- Fuzzy sets allow membership between 0 and 1.
- Membership function defines degree of belonging.
- A fuzzy set is described using ordered pairs.

# 1.5. Fuzzy Sets and Membership



**Question**: Define fuzzy set and membership function.

**Answer**:

A **fuzzy set** is a set in which each element has a degree of membership between 0 and 1.

A **membership function** assigns this degree to each element in the universe of discourse.

# 1.6.1 Chance versus Fuzziness

Chance and fuzziness are both related to uncertainty, but they are different.

**Chance**

Chance is related to probability or randomness.

Example:

`There is a 70% chance that it will rain tomorrow.`

This means the event may or may not happen.

**Fuzziness**

Fuzziness is related to vagueness.

Example:

`Today is 70% rainy.`

This means the concept "rainy" is partially true.

# 1.6.2 Chance versus Fuzziness: **Difference**

| Chance | Fuzziness |
|---|---|
| Based on randomness | Based on vagueness |
| Uses probability | Uses membership degree |
| Event either occurs or does not occur | Concept may be partially true |
| Example: 0.7 probability of rain | Example: 0.7 membership in rainy weather |

**Formula Comparison**

Probability: $P(A) \in [0,1]$

Membership: $\mu_A(x) \in [0,1]$

Although both range from 0 to 1, meanings are different.

**Worked Example**

Statement 1:

```
Probability that Ravi is tall = 0.8
```

This means we are uncertain whether Ravi is tall.

Statement 2:

```
Membership of Ravi in Tall = 0.8
```

This means Ravi is tall to degree 0.8.

# 1.6.3 Chance versus Fuzziness



**Quick Revision Points**

- Probability deals with randomness.
- Fuzziness deals with vagueness.
- Both use values from 0 to 1.
- Their interpretations are different.

# 1.6.4 Chance versus Fuzziness



**Question**: Differentiate chance and fuzziness with example.

**Answer**:

**Chance** refers to probability of occurrence of an event.

For example, there is a 0.7 probability of rain.

**Fuzziness** refers to degree of truth of a vague concept.

For example, weather is rainy to degree 0.7. Probability handles randomness, while fuzziness handles vagueness.

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Doubts?

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