LECTURE 03

21CSE451T

FUZZY LOGIC AND ITS APPLICATIONS

Unit-1 Introduction

1.13	Fuzzy Tolerance and Equivalence Relations
1.14	Value Assignments
1.15	Problems on fuzzy relation
1.16	Membership function – various forms
1.17	Fuzzification
1.18	Defuzzification to crisp sets

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SYLLABUS

Unit-1 - Introduction	***9 Hour***
The Case for Imprecision, The Utility of Fuzzy Systems, Limitations of Fuzzy Systems, Uncertainty and Information, Fuzzy Sets and Membership, Chance versus Fuzziness - Fuzzy Sets: Fuzzy Set Operations, Properties of Fuzzy Sets, Noninteractive Fuzzy Sets, Alternative Fuzzy Set Operations - Fuzzy Relations: Crisp Relations, Fuzzy Relations, Fuzzy Tolerance and Equivalence Relations, Value Assignments, Problems on fuzzy relation - Membership function – various forms –fuzzification – defuzzification to crisp sets.	
Unit-2 - Logic and Fuzzy Systems	***9 Hour***
classical logic, fuzzy logic, fuzzy systems – Development of Membership functions: membership value assignments, intuition, Inference, rank ordering – Automated Methods for Fuzzy Systems: Definitions, Batch Least Squares Algorithm, Recursive Least Squares Algorithm, Gradient Method, Learning From Example, Modified Learning From Example, Problems on logic and fuzzy systems	
Unit-3 - Rule-Base Reduction Methods	***9 Hour***
: Fuzzy Systems Theory and Rule Reduction, Singular Value Decomposition, Combs Method, SVD and Combs Method Examples, problems on SVD and Combs method for rapid inference - Decision Making with Fuzzy Information: Fuzzy Synthetic Evaluation, Fuzzy Ordering, Nontransitive Ranking, Preference and Consensus, Multiobjective Decision Making, Decision Making under Fuzzy States and Fuzzy Actions, problems on decision making with fuzzy information.	
Unit-4 - Fuzzy Classification and Pattern Recognition	***9 Hour***
Classification by Equivalence Relations, Cluster Analysis, Cluster Validity, c-Means Clustering, Fuzzy c-Means, Classification Metric, Similarity Relations from Clustering - Pattern Recognition: Feature Analysis, Partitions of the Feature Space, Single-Sample Identification, Multifeature Pattern Recognition, problems on fuzzy classification and pattern recognition, Case Study: Hand written character recognition using fuzzy logic.	
Unit-5 - Fuzzy Control Systems	***9 Hour***
Control System Design Problem, Control (Decision) Surface, Assumptions in a Fuzzy Control System Design, Simple Fuzzy Logic Controllers, Examples of Fuzzy Control System Design, Aircraft Landing Control Problem - Fuzzy Optimization - Fuzzy Linear Regression – problems on fuzzy optimization and regression, Case study: Robot Navigation using fuzzy logic.	

LECTURE 03 TOPICS

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1.13	Fuzzy Tolerance and Equivalence Relations
1.14	Value Assignments
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1.16	Membership function – various forms
1.17	Fuzzification
1.18	Defuzzification to crisp sets

1.13.1 Fuzzy Tolerance and Equivalence Relations

Fuzzy Tolerance Relation

A fuzzy tolerance relation is a fuzzy relation that satisfies:

1. Reflexivity
2. Symmetry

1. Reflexivity — every element is fully related to itself:

$R(x, x) = 1$ **for all x**

2. Symmetry — the relation between x and y is the same in both directions:

$R(x, y) = R(y, x)$ **for all x, y**

Example — "Similar Heights"

Consider people:

Alice (160 cm), Bob (163 cm), Carol (175 cm), Dave (190 cm).

Define R(x, y) = similarity based on how close their heights are:

R	A	B	C	D
A	1.0	0.9	0.5	0.0
B	0.9	1.0	0.6	0.1
C	0.5	0.6	1.0	0.5
D	0.0	0.1	0.5	1.0

1.13.1q Fuzzy Tolerance and Equivalence Relations

Example Problem — "Similar Heights"

Consider people:

A (160 cm), B (163 cm), C (175 cm), D (190 cm).

Check fuzzy tolerance

1.13.1a Fuzzy Tolerance and Equivalence Relations

Solution for checking the fuzzy tolerance relation on heights of A, B, C, D.

Step 1: Set up the data

Person	Height (cm)
A	160
B	163
C	175
D	190

1.13.1a Fuzzy Tolerance and Equivalence Relations

Step 2: Define the fuzzy membership function

A common way to build a fuzzy relation from a numeric attribute (like height) is:

$$\mu_R(x, y) = 1 - \frac{|h(x) - h(y)|}{\Delta_{max}}$$

where Δ_{max} is the largest height difference in the dataset.

1.13.1a Fuzzy Tolerance and Equivalence Relations

Pairwise differences:

Pair	|diff|
A,B	3
A,C	15
A,D	30
B,C	12
B,D	27
C,D	15

Largest difference = **30** (A vs D), so $\Delta_{max} = 30.$

1.13.1a Fuzzy Tolerance and Equivalence Relations

Step 3: Build the fuzzy relation matrix R

$$\mu_R(x, y) = 1 - \frac{|diff|}{30}$$

R	A	B	C	D
A	1.0	0.9	0.5	0.0
B	0.9	1.0	0.6	0.1
C	0.5	0.6	1.0	0.5
D	0.0	0.1	0.5	1.0

1.13.1a Fuzzy Tolerance and Equivalence Relations

R	A	B	C	D
A	1.0	0.9	0.5	0.0
B	0.9	1.0	0.6	0.1
C	0.5	0.6	1.0	0.5
D	0.0	0.1	0.5	1.0

Step 4: Check Reflexivity

$\mu_R(x, x) = 1$ for all x → diagonal is all 1's. ✅ **Reflexive**

Step 5: Check Symmetry

Matrix is symmetric across the diagonal (e.g., $\mu(A, B) = \mu(B, A) = 0.9$). ✅ **Symmetric**

Reflexive + Symmetric → R is a valid Fuzzy Tolerance Relation.

1.13.2q Fuzzy Tolerance and Equivalence Relations (Solved Problem)

Question:

Given fuzzy relation matrix:

$$R = \begin{bmatrix} 1 & 0.5 & 0.3 \\ 0.5 & 1 & 0.4 \\ 0.3 & 0.4 & 1 \end{bmatrix}$$

Check tolerance relation.

That is, prove:

$$\mu_R(x, x) = 1$$

$$\mu_R(x, y) = \mu_R(y, x)$$

1.13.2a Fuzzy Tolerance and Equivalence Relations (Solved Problem)

Question:

Given fuzzy relation matrix:

$$R = \begin{bmatrix} 1 & 0.5 & 0.3 \\ 0.5 & 1 & 0.4 \\ 0.3 & 0.4 & 1 \end{bmatrix}$$

Check tolerance relation.

Answer:

1. Reflexivity

Diagonal values:

1, 1, 1

$$\mu_R(x, x) = 1$$

So it is reflexive.

2. Symmetry

The matrix is symmetric because:

$R_{12}=R_{21}=0.5$

$R_{13}=R_{31}=0.3$

$R_{23}=R_{32}=0.4$

$$\mu_R(x, y) = \mu_R(y, x)$$

Therefore, it is a fuzzy tolerance relation.

1.13.3 Fuzzy Tolerance and Equivalence Relations

Fuzzy Equivalence Relation

A fuzzy equivalence relation satisfies:

$$\mu_R(x, x) = 1$$

1.Reflexivity [**R(x, x) = 1 for all x**]

$$\mu_R(x, y) = \mu_R(y, x)$$

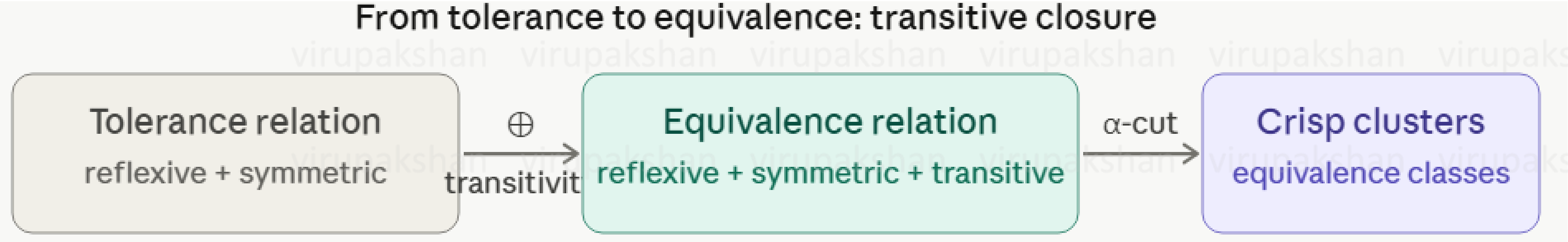
2.Symmetry [**R(x, y) = R(y, x) for all x, y**]

AND

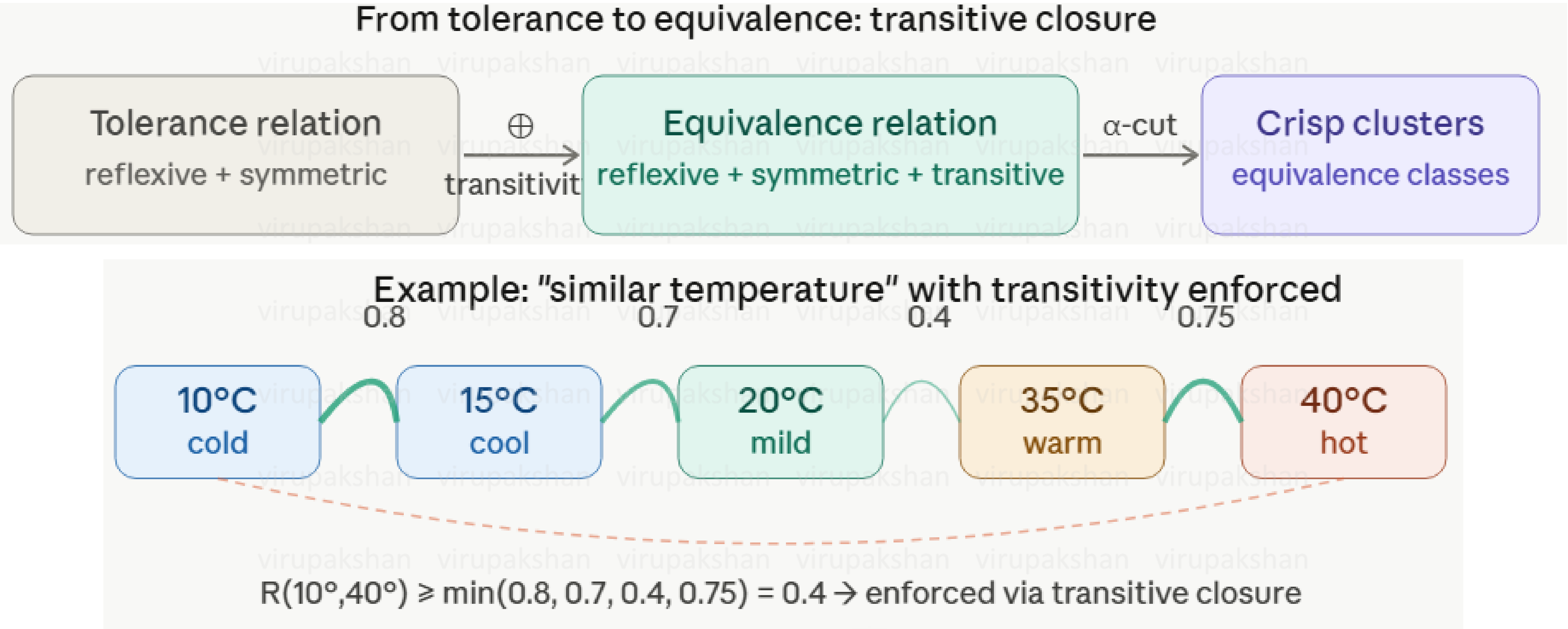
$$\mu_R(x, z) \geq \max_y \min[\mu_R(x, y), \mu_R(y, z)]$$

3. Max-min transitivity — for all x, y, z:

R(x, z) ≥ max over all y of [min (R(x, y), R(y, z))]



1.13.4 Fuzzy Tolerance and Equivalence Relations



1.13.5q Fuzzy Tolerance and Equivalence Relations

Consider the fuzzy relation:

	A	B	C
A	1	0.7	0.7
B	0.7	1	0.7
C	0.7	0.7	1

Verify "Fuzzy Equivalence Relation"

That is, prove:

$$\mu_R(x,x) = 1$$

$$\mu_R(x,y) = \mu_R(y,x)$$

$$\mu_R(x,z) \geq \max_y \min[\mu_R(x,y), \mu_R(y,z)]$$

	A	B	C
A	1	0.7	0.7
B	0.7	1	0.7
C	0.7	0.7	1

1.13.5a Fuzzy Tolerance and Equivalence Relations (1 page answer)

Consider the fuzzy relation:

	A	B	C
A	1	0.7	0.7
B	0.7	1	0.7
C	0.7	0.7	1

Verify "Fuzzy Equivalence Relation"

$$\mu_R(x,x) = 1$$

$$\mu_R(x,y) = \mu_R(y,x)$$

$$\mu_R(x,z) \geq \max_y \min[\mu_R(x,y), \mu_R(y,z)]$$

Verification

1) Reflexive

- R(A,A)=1
- R(B,B)=1
- R(C,C)=1

✓

2) Symmetric

- R(A,B)=R(B,A)=0.7
- R(A,C)=R(C,A)=0.7
- R(B,C)=R(C,B)=0.7

✓

3) Transitive

For example:

$$\min(R,(A,B)R(B,C)) = \min(0.7,0.7) = 0.7$$

and

$$R(A,C) = 0.7$$

Hence,

$$R(A,C) \geq \min(R,(A,B)R(B,C)) \ ✓$$

Therefore, **R is a Fuzzy Equivalence Relation**.

To verify whether the given fuzzy relation R is a **Fuzzy Equivalence Relation** (also known as a **similarity relation**), it must satisfy three core properties:

1. **Reflexivity**
2. **Symmetry**
3. **Max-Min Transitivity**

	A	B	C
A	1	0.7	0.7
B	0.7	1	0.7
C	0.7	0.7	1

That is, prove:

$$\mu_R(x, x) = 1$$

$$\mu_R(x, y) = \mu_R(y, x)$$

$$\mu_R(x, z) \geq \max_y \min[\mu_R(x, y), \mu_R(y, z)]$$

	A	B	C
A	1	0.7	0.7
B	0.7	1	0.7
C	0.7	0.7	1

Step 1: Check Reflexivity

We need R(x,x) = 1 for all elements.

- R(A,A) = 1 ✓
- R(B,B) = 1 ✓
- R(C,C) = 1 ✓

Reflexivity holds.

	A	B	C
A	1	0.7	0.7
B	0.7	1	0.7
C	0.7	0.7	1

$$\mu_R(x,x) = 1$$

	A	B	C
A	1	0.7	0.7
B	0.7	1	0.7
C	0.7	0.7	1

Step 2: Check Symmetry

We need R(x,y) = R(y,x) for all pairs.

- R(A,B) = 0.7 and R(B,A) = 0.7 ✓
- R(A,C) = 0.7 and R(C,A) = 0.7 ✓
- R(B,C) = 0.7 and R(C,B) = 0.7 ✓

	A	B	C
A	1	0.7	0.7
B	0.7	1	0.7
C	0.7	0.7	1

$$\mu_R(x,y) = \mu_R(y,x)$$

Symmetry holds (the matrix is symmetric about the diagonal).

1.13.5a Fuzzy Tolerance and Equivalence Relations

	A	B	C
A	1	0.7	0.7
B	0.7	1	0.7
C	0.7	0.7	1

Step 3: Check Max-Min Transitivity

This is the key test. We need:

$$R(x, z) \geq \max_y [\min(R(x, y), R(y, z))]$$

This is equivalent to computing $\mathbf{R} \circ \mathbf{R}$ (max-min composition of R with itself) and checking if $\mathbf{R} \circ \mathbf{R} \subseteq \mathbf{R}$ (i.e., every entry of R ∘ R is ≤ corresponding entry of R). For an equivalence relation, we actually need R ∘ R = R.

1.13.5a Fuzzy Tolerance and Equivalence Relations

	A	B	C
A	1	0.7	0.7
B	0.7	1	0.7
C	0.7	0.7	1

Compute (R ◦ R)(A,A):

$$\max[\min(1,1),\min(0.7,0.7),\min(0.7,0.7)] = \max[1,0.7,0.7] = 1$$

Compute (R ◦ R)(A,B):

$$\max[\min(R(A,A),R(A,B)),\min(R(A,B),R(B,B)),\min(R(A,C),R(C,B))]$$

$$= \max[\min(1,0.7),\min(0.7,1),\min(0.7,0.7)]$$

$$= \max[0.7,0.7,0.7] = 0.7$$

1.13.5a Fuzzy Tolerance and Equivalence Relations

	A	B	C
A	1	0.7	0.7
B	0.7	1	0.7
C	0.7	0.7	1

Compute (R ∘ R)(A,C):

$$\max[\min(R(A,A),R(A,C)),\min(R(A,B),R(B,C)),\min(R(A,C),R(C,C))]$$

$$= \max[\min(1,0.7),\min(0.7,0.7),\min(0.7,1)]$$

$$= \max[0.7,0.7,0.7] = 0.7$$

1.13.5a Fuzzy Tolerance and Equivalence Relations

	A	B	C
A	1	0.7	0.7
B	0.7	1	0.7
C	0.7	0.7	1

Compute (R ∘ R)(B,C):

$$\max[\min(R(B,A),R(A,C)),\min(R(B,B),R(B,C)),\min(R(B,C),R(C,C))]$$

$$=\max[\min(0.7,0.7),\min(1,0.7),\min(0.7,1)]$$

$$=\max[0.7,0.7,0.7]=0.7$$

	A	B	C
A	1	0.7	0.7
B	0.7	1	0.7
C	0.7	0.7	1

By symmetry, the remaining entries **(B,A), (C,A), (C,B), and the diagonal entries (B,B), (C,C)** follow the same pattern.

So the full composition matrix is:

$$R \circ R = \begin{pmatrix} 1 & 0.7 & 0.7 \\ 0.7 & 1 & 0.7 \\ 0.7 & 0.7 & 1 \end{pmatrix}$$

1.13.5a Fuzzy Tolerance and Equivalence Relations

	A	B	C
A	1	0.7	0.7
B	0.7	1	0.7
C	0.7	0.7	1

Comparing R∘R with R: they are exactly equal.

Since R ∘ R = R, transitivity holds for **every** triple (you can also verify directly:

for any x,z, R(x,z) = 0.7 or 1, and

max[min(R(x,y),R(y,z))]

never exceeds 0.7 when R(x,z)=0.7, and equals 1 only when x=z).

Transitivity holds.

1.13.5a Fuzzy Tolerance and Equivalence Relations

	A	B	C
A	1	0.7	0.7
B	0.7	1	0.7
C	0.7	0.7	1

Conclusion

Property	Status
Reflexivity	✅ Satisfied
Symmetry	✅ Satisfied
Max-Min Transitivity	✅ Satisfied

Since all three properties hold, **R is a valid fuzzy equivalence relation**

1.13.6 Fuzzy Tolerance and Equivalence Relations: Python code

python

```
R = [
    [1, 0.5, 0.3],
    [0.5, 1, 0.4],
    [0.3, 0.4, 1]
]

def is_reflexive(R):
    for i in range(len(R)):
        if R[i][i] != 1:
            return False
    return True
```

```
def is_symmetric(R):
    n = len(R)
    for i in range(n):
        for j in range(n):
            if R[i][j] != R[j][i]:
                return False
    return True

print("Reflexive:", is_reflexive(R))
print("Symmetric:", is_symmetric(R))
print("Tolerance relation:", is_reflexive(R) and is_symmetric(R))
```

1.13.7 Fuzzy Tolerance and Equivalence Relations: Summary

Fuzzy Tolerance Relation:

$\boxed{\text{Reflexive + Symmetric}}$

Fuzzy Equivalence Relation:

$\boxed{\text{Reflexive + Symmetric + Transitive}}$

- **Fuzzy tolerance** relation is reflexive and symmetric.
- **Fuzzy equivalence** relation is reflexive, symmetric, and transitive.
- Transitivity is usually checked using max-min composition.

1.14.1 Value Assignments: Meaning

Value assignment means assigning membership values to elements based on expert knowledge, observation, measurement, or mathematical functions.

It is the process of deciding how strongly an element belongs to a fuzzy set.

Example

For fuzzy set **Tall**:

Height in cm	Membership in Tall
150	0.0
160	0.2
170	0.5
180	0.8
190	1.0

1.14.2 Value Assignments: Methods of Value Assignment

1. Expert-Based Assignment

Experts assign values.

Example: A doctor assigns degree of fever based on temperature.

2. Data-Based Assignment

Membership values are calculated from collected data.

3. Formula-Based Assignment

A mathematical membership function is used.

Example:

$$\mu_{\text{Tall}}(h) = \begin{cases} 0, & h \leq 150 \\ \frac{h-150}{40}, & 150 < h < 190 \\ 1, & h \geq 190 \end{cases}$$

1.14.3 Value Assignments: Worked Example

Q) Find membership of height 170 cm in Tall.

Using:

$$\mu_{\text{Tall}}(h) = \frac{h - 150}{40}$$

A) For h =170:

$$\mu_{\text{Tall}}(170) = \frac{170 - 150}{40} = \frac{20}{40} = 0.5$$

1.14.4 Value Assignments: Q&A

Question: What is value assignment in fuzzy logic?

Answer:

Value assignment is the process of assigning membership values between 0 and 1 to elements of a fuzzy set. It can be done using expert opinion, collected data, or mathematical membership functions.

1.14.5 Value Assignments: Python code

python

```python
def tall_membership(height):
    if height <= 150:
        return 0
    elif height >= 190:
        return 1
    else:
        return (height - 150) / 40


height = 170
print("Membership in Tall:", tall_membership(height))
```

1.15.1 Problems on fuzzy relation

Max-Min Composition in Fuzzy Relations

Max-Min composition is the most important operation for **combining two fuzzy relations** to derive a new one.

It answers the question:

"If x is related to y, and y is related to z, how related are x and z?"

If:

- R is a relation from X to Y
- S is a relation from Y to Z

Then composition $T = R \circ S$ is:

$$\mu_T(x,z) = \max_y \min[\mu_R(x,y), \mu_S(y,z)]$$

1.15.2 Problems on fuzzy relation: Max-Min Composition

The core idea

Given two fuzzy relations:

- **R** on sets X × Y — "x is related to y with degree R(x,y)"
- **S** on sets Y × Z — "y is related to z with degree S(y,z)"

The composition **T = R ∘ S** on X × Z is computed as:

> **T(x, z) = max over all y of [min(R(x,y), S(y,z))]**

The logic is elegant:

- **min** — the strength of a *path* through y is limited by its weakest link
- **max** — take the *best* (strongest) path found among all possible y

1.15.3 Problems on fuzzy relation: Worked Problem

Given: $R = \begin{bmatrix} 0.2 & 0.7 \\ 0.5 & 1.0 \end{bmatrix}$ and $S = \begin{bmatrix} 0.6 & 0.3 \\ 0.4 & 0.8 \end{bmatrix}$ Find: $T = R \circ S$

Step-by-Step Solution

$$\mu_T(x, z) = \max_y \min[\mu_R(x, y), \mu_S(y, z)]$$

Calculate T_{11}

Calculate T_{12}

Calculate T_{21}

Calculate T_{22}

T(x, y) = **max {** min(R(x,1), S(1,y)) **,** min(R(x,2), S(2,y)) **}**

1.15.3 Problems on fuzzy relation: Worked Problem

Given: $R = \begin{bmatrix} 0.2 & 0.7 \\ 0.5 & 1.0 \end{bmatrix}$ and $S = \begin{bmatrix} 0.6 & 0.3 \\ 0.4 & 0.8 \end{bmatrix}$ Find: $T = R \circ S$

Step-by-Step Solution

T(x, y) = **max {** min(R(x,1), S(1,y)) **,** min(R(x,2), S(2,y)) **}**

Calculate T_{11}

T_{11}=max[min(0.2,0.6),min(0.7,0.4)]

=max[0.2,0.4]=0.4

Calculate T_{12}

T_{12}=max[min(0.2,0.3),min(0.7,0.8)]

=max[0.2,0.7]=0.7

Calculate T_{21}

T_{21}=max[min(0.5,0.6),min(1.0,0.4)]

=max[0.5,0.4]=0.5

Calculate T_{22}

T_{22}=max[min(0.5,0.3),min(1.0,0.8)]

=max[0.3,0.8]=0.8

Final answer: $T = \begin{bmatrix} 0.4 & 0.7 \\ 0.5 & 0.8 \end{bmatrix}$

$$R = \begin{bmatrix} 0.2 & 0.7 \\ 0.5 & 1.0 \end{bmatrix} \quad S = \begin{bmatrix} 0.6 & 0.3 \\ 0.4 & 0.8 \end{bmatrix}$$

T(x, y) = max over z of [min(R(x, z), S(z, y))]

T(1,1) — row 1 of R × col 1 of S

z=1: min(R(1,1), S(1,1)) = min(0.2, 0.6) = 0.2
z=2: min(R(1,2), S(2,1)) = min(0.7, 0.4) = 0.4

max(0.2, 0.4) = **0.4**

T(1,2) — row 1 of R × col 2 of S

z=1: min(R(1,1), S(1,2)) = min(0.2, 0.3) = 0.2
z=2: min(R(1,2), S(2,2)) = min(0.7, 0.8) = 0.7

max(0.2, 0.7) = **0.7**

T(2,1) — row 2 of R × col 1 of S

z=1: min(R(2,1), S(1,1)) = min(0.5, 0.6) = 0.5
z=2: min(R(2,2), S(2,1)) = min(1.0, 0.4) = 0.4

max(0.5, 0.4) = **0.5**

T(2,2) — row 2 of R × col 2 of S

z=1: min(R(2,1), S(1,2)) = min(0.5, 0.3) = 0.3
z=2: min(R(2,2), S(2,2)) = min(1.0, 0.8) = 0.8

max(0.3, 0.8) = **0.8**

1.15.3b Problems on fuzzy relation: Solution (simple)

$$R = \begin{bmatrix} 0.2 & 0.7 \\ 0.5 & 1.0 \end{bmatrix} \quad S = \begin{bmatrix} 0.6 & 0.3 \\ 0.4 & 0.8 \end{bmatrix}$$

$$\mu_T(x,z) = \max_y \min[\mu_R(x,y), \mu_S(y,z)]$$

Calculate T_{11}

T_{11}=max[min(0.2,0.6),min(0.7,0.4)]

=max[0.2,0.4]=**0.4**

Calculate T_{12}

T_{12}=max[min(0.2,0.3),min(0.7,0.8)]

=max[0.2,0.7]=**0.7**

Calculate T_{21}

T_{21}=max[min(0.5,0.6),min(1.0,0.4)]

=max[0.5,0.4]=**0.5**

Calculate T_{22}

T_{22}=max[min(0.5,0.3),min(1.0,0.8)]

=max[0.3,0.8]=**0.8**

Final answer: $T = \begin{bmatrix} 0.4 & 0.7 \\ 0.5 & 0.8 \end{bmatrix}$

1.16.0 Membership function – various forms

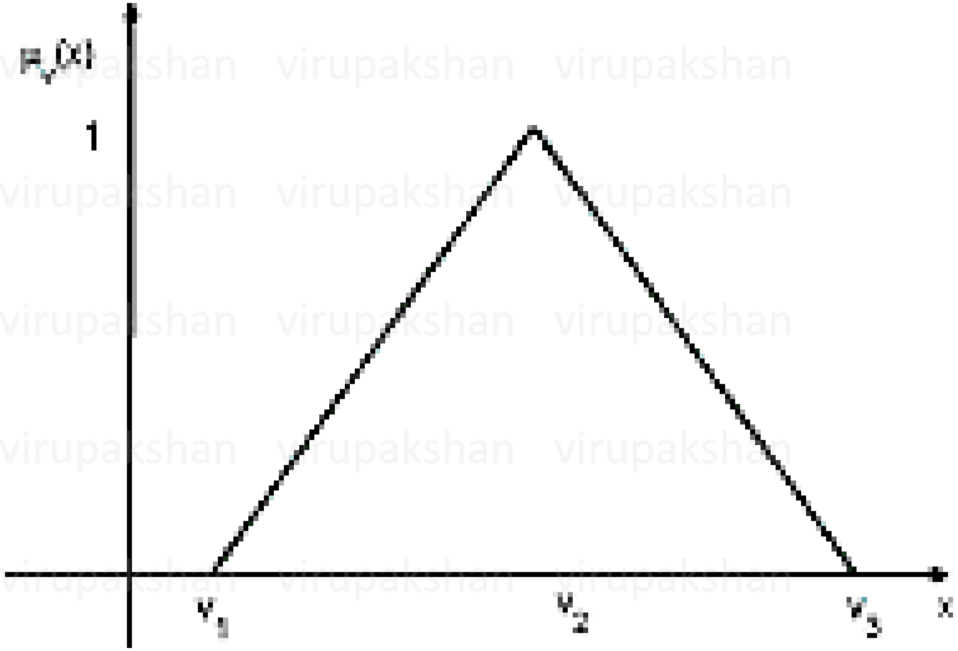
A **membership function** defines how each input value maps to a membership value between 0 and 1.

Common membership function shapes are:

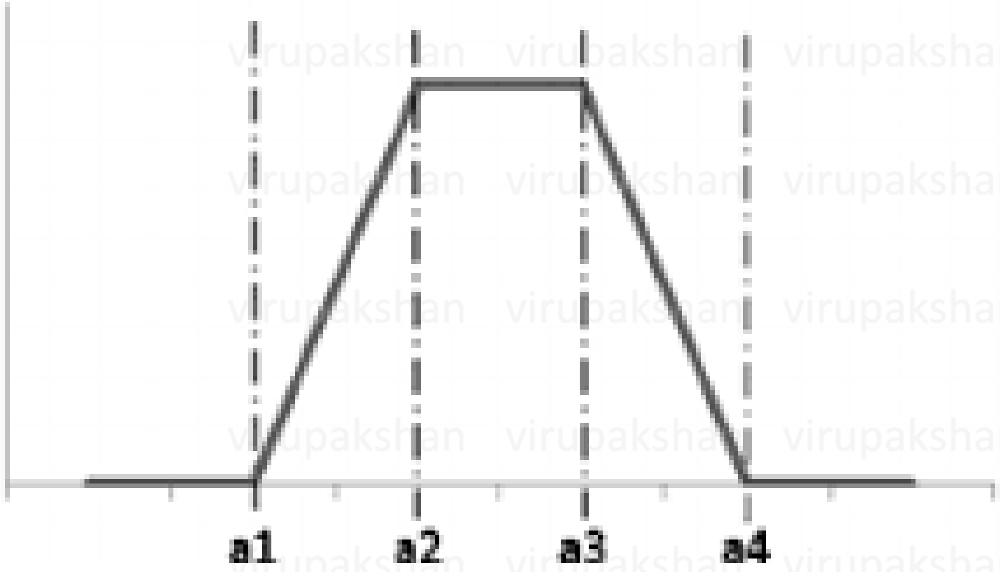
1. Triangular
2. Trapezoidal
3. Gaussian
4. Sigmoid
5. Singleton
6. Bell-shaped

1.16.1 Membership function – Triangular

$$\mu_v(x) = \begin{cases} 0, & x < v_1 \\ \dfrac{x-v_1}{v_2-v_1}, & v_1 \leq x \leq v_2 \\ \dfrac{v_3-x}{v_3-v_2}, & v_2 \leq x \leq v_3 \\ 0, & x > v_3 \end{cases}$$



1.16.2 Membership function – Trapezoidal



$$f(x; a1, a2, a3, a4) = \begin{cases} 0, x \leq a1 \\ \dfrac{x - a1}{a2 - a1}, \\ \dfrac{a4 - x}{a4 - a3}, \\ 0, a4 \leq x \end{cases}$$

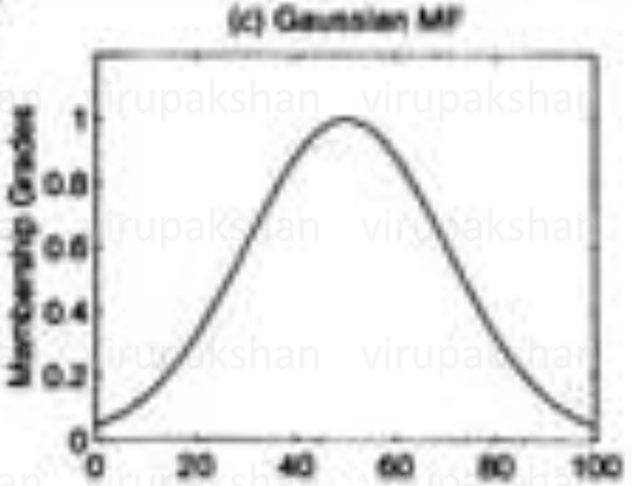
With $a1 \leq x \leq a2$ and $a3 \leq x \leq a4$

1.16.3 Membership function – Gaussian

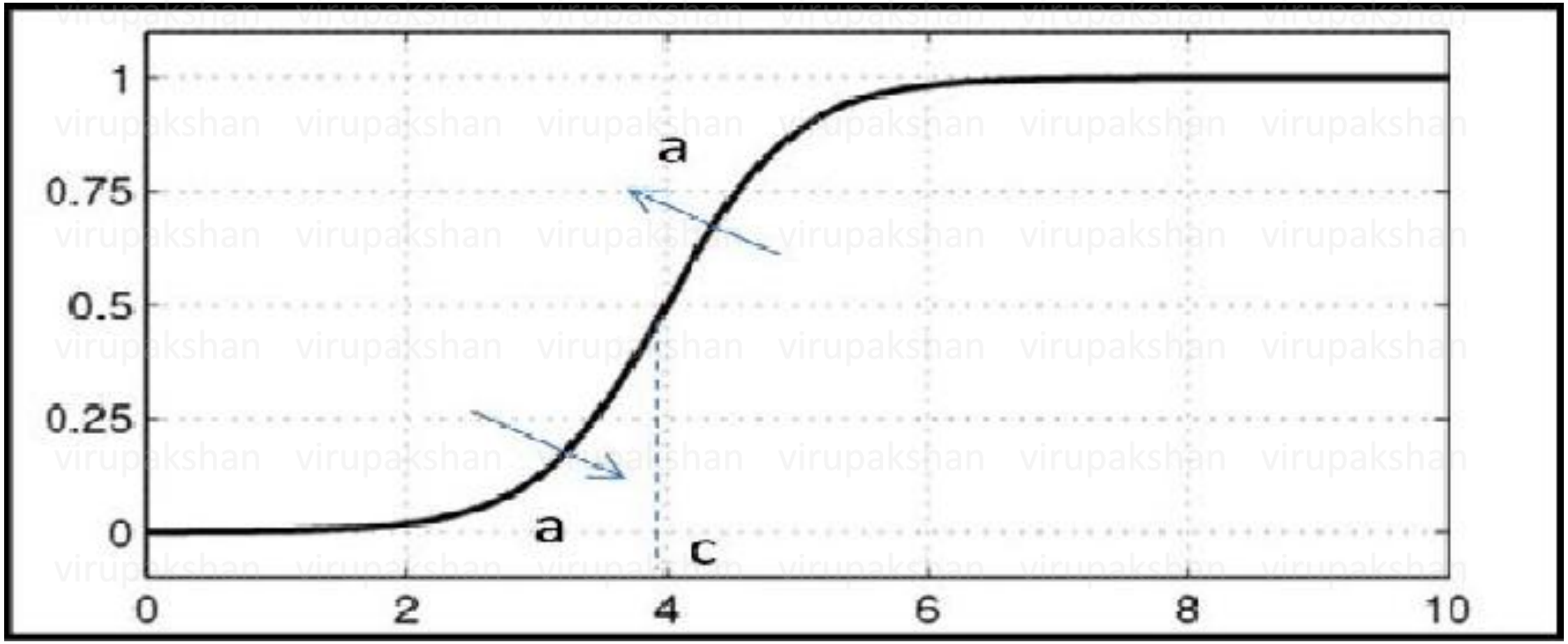
- It is defined by 2 parameters {c,σ}
- C is the center, σ is the MFs width.

$$\text{gaussian}(x; c, \sigma) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2}.$$

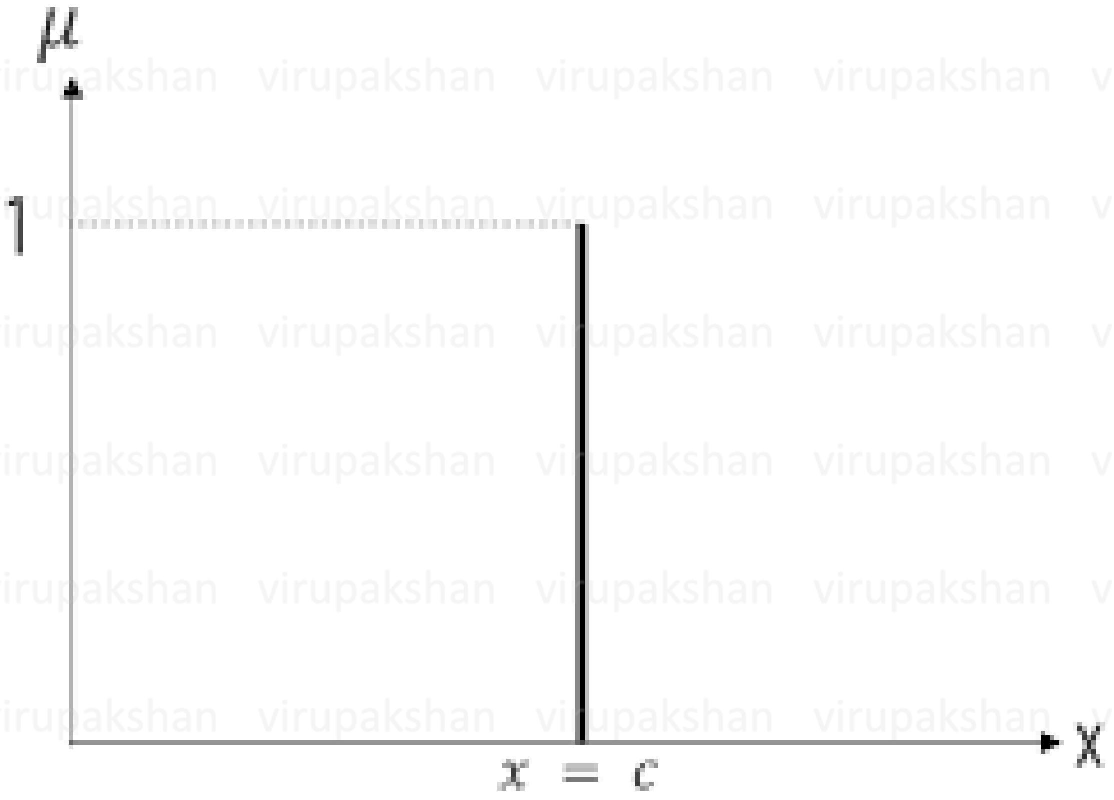
- Ex. Gaussian(x,50,20)



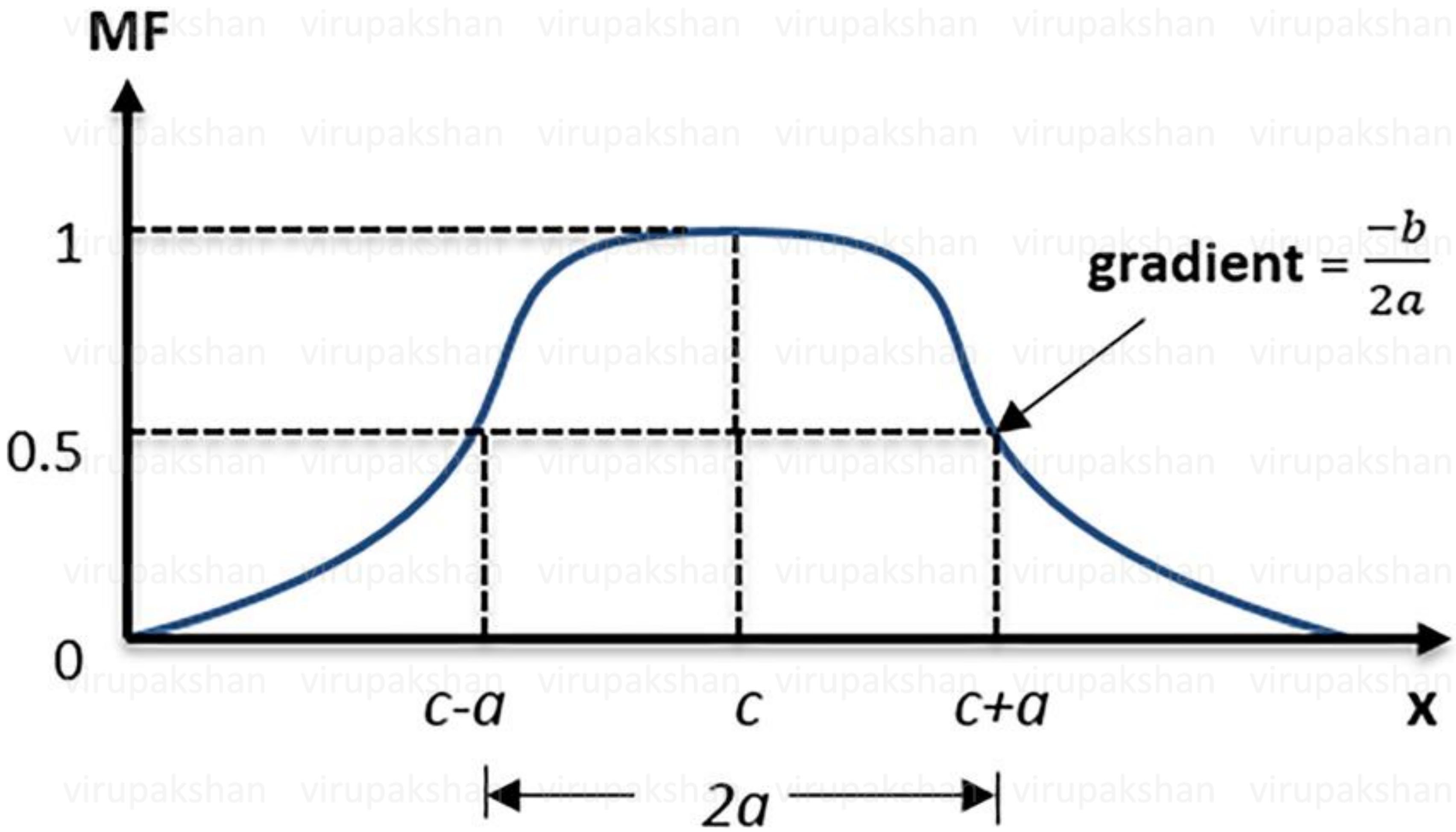
1.16.4 Membership function – Sigmoid



1.16.5 Membership function – Singleton



1.16.6 Membership function – Bell-shaped



1.16.7 Membership function – Q&A

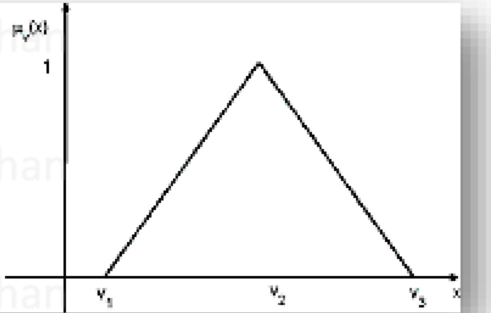
Question: Write the triangular membership function and calculate membership for x =15, a =10, b =20, c =30.

Answer:

For 10 <x ≤20:

$$\mu(x) = \frac{x-a}{b-a}$$

$$\mu_v(x) = \begin{cases} 0, & x < v_1 \\ \frac{x-v_1}{v_2-v_1}, & v_1 \le x \le v_2 \\ \frac{v_3-x}{v_3-v_2}, & v_2 \le x \le v_3 \\ 0, & x > v_3 \end{cases}$$



So:

$$\mu(15) = \frac{15-10}{20-10} = 0.5$$

1.17.1 Fuzzification

Meaning

Fuzzification is the process of converting a crisp input into fuzzy membership values.

Example:

Crisp input:

Temperature = 28°C

Fuzzy output:

Cool = 0.3

Warm = 0.8

Hot = 0.2

Fuzzification Process

Crisp Input

↓

Membership Functions

↓

Fuzzy Values

1.17.2 Fuzzification – Why?

Why Fuzzification Is Needed?

Computers receive numerical values, but fuzzy rules use linguistic terms.

Example rule:

```
IF temperature is warm THEN fan speed is medium.
```

To apply this rule, the crisp temperature must be converted into fuzzy values.

1.17.3 Fuzzification - **Example**

Suppose temperature = 30°C.

Membership functions give:

Fuzzy Set	**Membership**
Cool	0.1
Warm	0.6
Hot	0.8

Therefore, the crisp value 30°C is fuzzified as:

```
Cool = 0.1, Warm = 0.6, Hot = 0.8
```

1.17.4 Fuzzification – Q&A

Question: What is fuzzification? Explain with example.

Answer:
Fuzzification is the process of converting a crisp numerical input into fuzzy membership values.

For example, if temperature is 32°C, it may be represented as *Warm = 0.8 and Hot = 0.2* using membership functions.

1.18.1 Defuzzification to crisp sets - **Meaning**

Defuzzification is the process of converting fuzzy output into a crisp output value.

After fuzzy rules are applied, the output is usually fuzzy.

Example: **Fan speed is medium to degree 0.6 and high to degree 0.8.**

But a machine needs a crisp value such as:

Fan speed = 75%

So defuzzification is needed.

Fuzzy Output
↓
Defuzzification
↓
Crisp Output

1.18.2 Defuzzification to crisp sets - **Methods**

Common Defuzzification Methods

1. Centroid method
2. Bisector method
3. Mean of maximum
4. Smallest of maximum
5. Largest of maximum
6. Weighted average method

1.18.3 Defuzzification to crisp sets – **Centroid Method**

The centroid method finds the center of area of the fuzzy output.

Formula:

$$z^* = \frac{\int z\mu(z)\,dz}{\int \mu(z)\,dz}$$

For discrete values:

$$z^* = \frac{\sum z_i \mu(z_i)}{\sum \mu(z_i)}$$

1.18.4 Defuzzification to crisp sets – **Centroid Method Example**

Suppose a fuzzy system gives:

Fan Speed	Membership
40	0.2
60	0.5
80	0.8
100	0.4

$$z^* = \frac{\sum z_i \mu(z_i)}{\sum \mu(z_i)}$$

Using centroid method:

$$z^* = \frac{40(0.2) + 60(0.5) + 80(0.8) + 100(0.4)}{0.2 + 0.5 + 0.8 + 0.4}$$

$$z^* = \frac{8 + 30 + 64 + 40}{1.9}$$

$$z^* = \frac{142}{1.9} = 74.74$$

So crisp fan speed is approximately:

74.74%

LECTURE 03 TOPICS

Unit-1 - Introduction	**9 Hour**
The Case for Imprecision. The Utility of Fuzzy Systems. Limitations of Fuzzy Systems. Uncertainty and Information. Fuzzy Sets and Membership. Chance versus Fuzziness - Fuzzy Sets: Fuzzy Set Operations. Properties of Fuzzy Sets. Noninteractive Fuzzy Sets. Alternative Fuzzy Set Operations - Fuzzy Relations: Crisp Relations. Fuzzy Relations. Fuzzy Tolerance and Equivalence Relations. Value Assignments. Problems on fuzzy relation - Membership function – various forms –fuzzification – defuzzification to crisp sets.	
Unit-2 - Logic and Fuzzy Systems	**9 Hour**
classical logic. fuzzy logic. fuzzy systems – Development of Membership functions: membership value assignments. intuition. Inference. rank ordering – Automated Methods for Fuzzy Systems: Definitions. Batch Least Squares Algorithm. Recursive Least Squares Algorithm. Gradient Method. Learning From Example. Modified Learning From Example. Problems on logic and fuzzy systems	
Unit-3 - Rule-Base Reduction Methods	**9 Hour**
: Fuzzy Systems Theory and Rule Reduction. Singular Value Decomposition. Combs Method. SVD and Combs Method Examples. problems on SVD and Combs method for rapid inference - Decision Making with Fuzzy Information: Fuzzy Synthetic Evaluation. Fuzzy Ordering. Nontransitive Ranking. Preference and Consensus. Multiobjective Decision Making. Decision Making under Fuzzy States and Fuzzy Actions. problems on decision making with fuzzy information.	
Unit-4 - Fuzzy Classification and Pattern Recognition	**9 Hour**
Classification by Equivalence Relations. Cluster Analysis. Cluster Validity. c-Means Clustering. Fuzzy c-Means. Classification Metric. Similarity Relations from Clustering - Pattern Recognition: Feature Analysis. Partitions of the Feature Space. Single-Sample Identification. Multifeature Pattern Recognition. problems on fuzzy classification and pattern recognition. Case Study: Hand written character recognition using fuzzy logic.	
Unit-5 - Fuzzy Control Systems	**9 Hour**
Control System Design Problem. Control (Decision) Surface. Assumptions in a Fuzzy Control System Design. Simple Fuzzy Logic Controllers. Examples of Fuzzy Control System Design. Aircraft Landing Control Problem - Fuzzy Optimization - Fuzzy Linear Regression – problems on fuzzy optimization and regression. Case study: Robot Navigation using fuzzy logic.	

1.13	Fuzzy Tolerance and Equivalence Relations
1.14	Value Assignments
1.15	Problems on fuzzy relation
1.16	Membership function – various forms
1.17	Fuzzification
1.18	Defuzzification to crisp sets

LECTURE 03

21CSE451T

FUZZY LOGIC AND ITS APPLICATIONS

Doubts?

THANK YOU!

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