LECTURE 04

21CSE451T

FUZZY LOGIC AND ITS APPLICATIONS

Unit 2: Logic and Fuzzy Systems

2.1	Classical logic
2.2	Fuzzy logic
2.3	Fuzzy systems
2.4	Membership value assignments
2.5	Intuition

Virupakshan K

+91-9840847872 | VIRU@KART.LA

21CSE451T FUZZY LOGIC AND ITS APPLICATIONS
SYLLABUS

Unit-1 - Introduction	***9 Hour***
The Case for Imprecision, The Utility of Fuzzy Systems, Limitations of Fuzzy Systems, Uncertainty and Information, Fuzzy Sets and Membership, Chance versus Fuzziness - Fuzzy Sets: Fuzzy Set Operations, Properties of Fuzzy Sets, Noninteractive Fuzzy Sets, Alternative Fuzzy Set Operations - Fuzzy Relations: Crisp Relations, Fuzzy Relations, Fuzzy Tolerance and Equivalence Relations, Value Assignments, Problems on fuzzy relation - Membership function – various forms –fuzzification – defuzzification to crisp sets.	
Unit-2 - Logic and Fuzzy Systems	***9 Hour***
classical logic, fuzzy logic, fuzzy systems – Development of Membership functions: membership value assignments, intuition, Inference, rank ordering – Automated Methods for Fuzzy Systems: Definitions, Batch Least Squares Algorithm, Recursive Least Squares Algorithm, Gradient Method, Learning From Example, Modified Learning From Example, Problems on logic and fuzzy systems	
Unit-3 - Rule-Base Reduction Methods	***9 Hour***
: Fuzzy Systems Theory and Rule Reduction, Singular Value Decomposition, Combs Method, SVD and Combs Method Examples, problems on SVD and Combs method for rapid inference - Decision Making with Fuzzy Information: Fuzzy Synthetic Evaluation, Fuzzy Ordering, Nontransitive Ranking, Preference and Consensus, Multiobjective Decision Making, Decision Making under Fuzzy States and Fuzzy Actions, problems on decision making with fuzzy information.	
Unit-4 - Fuzzy Classification and Pattern Recognition	***9 Hour***
Classification by Equivalence Relations, Cluster Analysis, Cluster Validity, c-Means Clustering, Fuzzy c-Means, Classification Metric, Similarity Relations from Clustering - Pattern Recognition: Feature Analysis, Partitions of the Feature Space, Single-Sample Identification, Multifeature Pattern Recognition, problems on fuzzy classification and pattern recognition, Case Study: Hand written character recognition using fuzzy logic.	
Unit-5 - Fuzzy Control Systems	***9 Hour***
Control System Design Problem, Control (Decision) Surface, Assumptions in a Fuzzy Control System Design, Simple Fuzzy Logic Controllers, Examples of Fuzzy Control System Design, Aircraft Landing Control Problem - Fuzzy Optimization - Fuzzy Linear Regression – problems on fuzzy optimization and regression, Case study: Robot Navigation using fuzzy logic.	

Unit 2: Logic and Fuzzy Systems

#	Topic
2.1	Classical logic
2.2	Fuzzy logic
2.3	Fuzzy systems
2.4	Membership value assignments
2.5	Intuition
2.6	Inference
2.7	Rank ordering
2.8	Definitions
2.9	Batch Least Squares Algorithm
2.10	Recursive Least Squares Algorithm
2.11	Gradient Method
2.12	Learning From Example
2.13	Modified Learning From Example
2.14	Problems on logic and fuzzy systems

Lecture 04 Topics

2.1	Classical logic
2.2	Fuzzy logic
2.3	Fuzzy systems
2.4	Membership value assignments
2.5	Intuition

2.1 Classical logic

Classical logic is a two-valued logic system where every statement is either:

- **True**
- **False**

There is no middle value.

Problem with Classical Logic

Classical logic is too strict.

For example:

- *29.9°C is considered not hot*
- *30°C is considered hot*

But in real life, 29.9°C and 30°C feel almost the same.

This limitation led to the development of **fuzzy logic**.

Example

Statement:

"The temperature is hot."

In classical logic, we may define:

- If temperature ≥ 30°C, then hot = true
- If temperature < 30°C, then hot = false

So:

Temperature	Classical Decision
25°C	Not hot
29°C	Not hot
30°C	Hot
31°C	Hot

2.2.1 Fuzzy logic

Fuzzy logic is a many-valued logic system where truth values can lie between 0 and 1.

$$Truth\ value \in [0,1]$$

This means a statement can be:

- Completely false: 0
- Partially true: between 0 and 1
- Completely true: 1

Example

Statement:

"The temperature is hot."

In fuzzy logic:

Temperature	Degree of Hotness
20°C	0.0
25°C	0.3
30°C	0.7
35°C	1.0

So instead of saying 30°C is simply hot, fuzzy logic says:

30°C is hot to degree 0.7.

2.2.2 Difference Between Classical Logic and Fuzzy Logic

Feature	Classical Logic	Fuzzy Logic
Truth values	0 or 1	Between 0 and 1
Boundary	Sharp	Smooth
Reasoning	Exact	Approximate
Example	*Hot or not hot*	*Partially hot*
Suitable for	Digital systems	Human-like reasoning

2.3.1 Fuzzy systems

A **fuzzy system** is a system that uses fuzzy logic to map inputs to outputs.

It is useful when:

- The system is complex
- Mathematical modeling is difficult
- Human knowledge is available
- Input information is uncertain or vague

Basic Structure of a Fuzzy System

Crisp Input
↓
Fuzzification
↓
Rule Base
↓
Inference Engine
↓
Defuzzification
↓
Crisp Output

2.3.2 Fuzzy systems: Components of a Fuzzy System

1. Fuzzification

Converts crisp input into fuzzy values.

Example:

Input temperature = 28°C

Fuzzy values:

- Cool = 0.2
- Warm = 0.7
- Hot = 0.4

2. Rule Base

Contains IF-THEN rules.

Example:

```
IF temperature is hot THEN fan speed is high.
IF temperature is warm THEN fan speed is medium.
IF temperature is cool THEN fan speed is low.
```

3. Inference Engine

Applies fuzzy rules to generate fuzzy output.

4. Defuzzification

Converts fuzzy output into crisp output.

Example:

Fuzzy fan speed output is converted into:

```
Fan speed = 75 rpm
```

2.3.3 Fuzzy systems: Diagram of Fuzzy Controller


```
            +----------------+
Input →     | Fuzzification  |
            +----------------+
                    ↓
            +----------------+
            | Rule Base      |
            +----------------+
                    ↓
            +----------------+
            | Inference      |
            | Engine         |
            +----------------+
                    ↓
            +----------------+
Output ←    | Defuzzification|
            +----------------+
```

2.3.4.Q Fuzzy systems: Worked Example

Question: A fuzzy controller for a washing machine uses three fuzzy sets for "Dirtiness" (Low, Medium, High) and three for "Wash Time" (Short, Medium, Long).

(a) Construct a fuzzy rule base (at least 5 rules) relating Dirtiness to Wash Time.

(b) Using your rules, analyze how the wash time would be determined when Dirtiness = 0.7 (Medium) and 0.4 (High).

(c) Comment on how fuzzy reasoning provides smoother control than crisp logic in this case.

2.3.4.A.1 Fuzzy systems: Worked Example

Given

A fuzzy controller for a washing machine uses:

- **Dirtiness** = {Low, Medium, High}
- **Wash Time** = {Short, Medium, Long}

Input:

- μ(Medium Dirtiness) = 0.7
- μ(High Dirtiness) = 0.4

2.3.4.A.2 Fuzzy systems: Worked Example

(a) Fuzzy Rule Base (At least 5 Rules)

A suitable fuzzy rule base is:

Rule No.	IF Dirtiness is	THEN Wash Time is
R1	Low	Short
R2	Medium	Medium
R3	High	Long
R4	Low OR Medium	Medium
R5	Medium OR High	Long

(Any reasonable rule base relating dirtiness and wash time is acceptable.)

2.3.4.A.3 Fuzzy systems: Worked Example

(b) Determine the Wash Time

Step 1: Fuzzified Input

Given:

$$\mu_{Medium} = 0.7$$

$$\mu_{High} = 0.4$$

$$\mu_{Low} = 0$$

2.3.4.A.4 Fuzzy systems: Worked Example

Step 2: Rule Evaluation

Rule R1

IF Dirtiness = Low → Wash Time = Short

$$\alpha_1 = 0$$

Output:

$$Short = 0$$

Rule R2

IF Dirtiness = Medium → Wash Time = Medium

$$\alpha_2 = 0.7$$

Output:

$$Medium = 0.7$$

Rule R3

IF Dirtiness = High → Wash Time = Long

$$\alpha_3 = 0.4$$

Output:

$$Long = 0.4$$

Rule R4

IF Dirtiness = Low OR Medium → Wash Time = Medium

Using OR = MAX:

$$\alpha_4 = \max(0, 0.7) = 0.7$$

Output:

$$Medium = 0.7$$

Rule R5

IF Dirtiness = Medium OR High → Wash Time = Long

$$\alpha_5 = \max(0.7, 0.4) = 0.7$$

Output:

$$Long = 0.7$$

2.3.4.A.5 Fuzzy systems: Worked Example

Step 3: Aggregate Outputs

Using MAX aggregation:

For **Short**

$$\mu_{Short} = 0$$

For **Medium**

$$\mu_{Medium} = \max(0.7, 0.7) = 0.7$$

For **Long**

$$\mu_{Long} = \max(0.4, 0.7) = 0.7$$

Hence,

$$Wash\ Time = \{(Short, 0), (Medium, 0.7), (Long, 0.7)\}$$

2.3.4.A.6 Fuzzy systems: Worked Example

Final Decision

The washing machine should select a wash time that is a combination of **Medium** and **Long**, both activated with membership value **0.7**.

Therefore:

$$\boxed{\text{Recommended Wash Time} = \text{Between Medium and Long}}$$

(After defuzzification, the crisp wash time would lie between the centers of the Medium and Long membership functions.)

2.3.4.A.7 Fuzzy systems: Worked Example

(c) Why Fuzzy Reasoning Gives Smoother Control

Crisp Logic

A conventional controller may use:

- Dirtiness < 0.5 → Medium Wash
- Dirtiness ≥ 0.5 → Long Wash

Thus a tiny change from 0.49 to 0.50 causes a sudden jump in wash time.

Fuzzy Logic

For the given input:

$$Medium = 0.7, \quad High = 0.4$$

Both rules contribute simultaneously.

Output becomes:

$$Medium = 0.7, \quad Long = 0.7$$

Thus the wash time changes **gradually** rather than abruptly.

Advantages of Fuzzy Reasoning

1. Handles uncertainty and imprecise inputs.
2. Produces smooth transitions between control actions.
3. Mimics human decision making.
4. Avoids sudden changes in machine operation.
5. Improves washing efficiency and user comfort.

2.3.4.A.8 Fuzzy systems: Worked Example

Conclusion

For the input:

$$\mu(Medium) = 0.7, \quad \mu(High) = 0.4$$

the inferred wash time is

$$\boxed{\{(Medium, 0.7), (Long, 0.7)\}}$$

which corresponds to a wash duration **between Medium and Long**, demonstrating the smooth control achieved by fuzzy logic.

2.4 Membership value assignments

A **membership value** tells how strongly an element belongs to a fuzzy set.

It is represented by: $\mu_A(x)$ where: $0 \le \mu_A(x) \le 1$

Common Membership Functions

1. Triangular Membership Function

$$\mu_A(x) = \begin{cases} 0, & x \le a \\ \frac{x-a}{b-a}, & a < x \le b \\ \frac{c-x}{c-b}, & b < x < c \\ 0, & x \ge c \end{cases}$$

2. Trapezoidal Membership Function

$$\mu_A(x) = \begin{cases} 0, & x \le a \\ \frac{x-a}{b-a}, & a < x \le b \\ 1, & b < x \le c \\ \frac{d-x}{d-c}, & c < x < d \\ 0, & x \ge d \end{cases}$$

3. Gaussian Membership Function

$$\mu_A(x) = e^{-\frac{(x-c)^2}{2\sigma^2}}$$

2.5.1 Intuition: Meaning

In fuzzy logic, **intuition** means using human reasoning and common sense to define fuzzy rules and membership functions.

Humans often think in linguistic terms such as:

- Low, Medium, High
- Hot, Cold
- Fast, Slow
- Large, Small

Fuzzy logic converts such human ideas into mathematical models.

Example

A person may say:

```
If the room is very hot, increase the fan speed.
```

This is not a precise mathematical rule, but it is useful. Fuzzy logic allows us to represent this kind of reasoning.

2.5.2 Intuition: Intuitive Rule Example

For air conditioning:

IF temperature is high AND humidity is high THEN cooling is strong.

Here:

- Temperature is high
- Humidity is high
- Cooling is strong

are fuzzy concepts.

2.5.3 Intuition: Importance

Intuition helps in:

- Designing membership functions
- Creating fuzzy rules
- Handling vague information
- Modeling human decision-making

Lecture 5 and Lecture 6

2.6 Inference
2.7 Rank ordering
2.8 Definitions
2.9 Batch Least Squares Algorithm
2.10 Recursive Least Squares Algorithm

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2.11 Gradient Method
2.12 Learning From Example
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Doubts?

THANK YOU!

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