LECTURE 06

21CSE451T

FUZZY LOGIC AND ITS APPLICATIONS

Unit 2: Logic and Fuzzy Systems

2.11	Gradient Method
2.12	Learning From Example
2.13	Modified Learning From Example
2.14	Problems on logic and fuzzy systems

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21CSE451T FUZZY LOGIC AND ITS APPLICATIONS
SYLLABUS

Unit-1 - Introduction	***9 Hour***
The Case for Imprecision, The Utility of Fuzzy Systems, Limitations of Fuzzy Systems, Uncertainty and Information, Fuzzy Sets and Membership, Chance versus Fuzziness - Fuzzy Sets: Fuzzy Set Operations, Properties of Fuzzy Sets, Noninteractive Fuzzy Sets, Alternative Fuzzy Set Operations - Fuzzy Relations: Crisp Relations, Fuzzy Relations, Fuzzy Tolerance and Equivalence Relations, Value Assignments, Problems on fuzzy relation - Membership function – various forms –fuzzification – defuzzification to crisp sets.	
Unit-2 - Logic and Fuzzy Systems	***9 Hour***
classical logic, fuzzy logic, fuzzy systems – Development of Membership functions: membership value assignments, intuition, Inference, rank ordering – Automated Methods for Fuzzy Systems: Definitions, Batch Least Squares Algorithm, Recursive Least Squares Algorithm, Gradient Method, Learning From Example, Modified Learning From Example, Problems on logic and fuzzy systems	
Unit-3 - Rule-Base Reduction Methods	***9 Hour***
: Fuzzy Systems Theory and Rule Reduction, Singular Value Decomposition, Combs Method, SVD and Combs Method Examples, problems on SVD and Combs method for rapid inference - Decision Making with Fuzzy Information: Fuzzy Synthetic Evaluation, Fuzzy Ordering, Nontransitive Ranking, Preference and Consensus, Multiobjective Decision Making, Decision Making under Fuzzy States and Fuzzy Actions, problems on decision making with fuzzy information.	
Unit-4 - Fuzzy Classification and Pattern Recognition	***9 Hour***
Classification by Equivalence Relations, Cluster Analysis, Cluster Validity, c-Means Clustering, Fuzzy c-Means, Classification Metric, Similarity Relations from Clustering - Pattern Recognition: Feature Analysis, Partitions of the Feature Space, Single-Sample Identification, Multifeature Pattern Recognition, problems on fuzzy classification and pattern recognition, Case Study: Hand written character recognition using fuzzy logic.	
Unit-5 - Fuzzy Control Systems	***9 Hour***
Control System Design Problem, Control (Decision) Surface, Assumptions in a Fuzzy Control System Design, Simple Fuzzy Logic Controllers, Examples of Fuzzy Control System Design, Aircraft Landing Control Problem - Fuzzy Optimization - Fuzzy Linear Regression – problems on fuzzy optimization and regression, Case study: Robot Navigation using fuzzy logic.	

Unit 2: Logic and Fuzzy Systems

#	Topic
2.1	Classical logic
2.2	Fuzzy logic
2.3	Fuzzy systems
2.4	Membership value assignments
2.5	Intuition
2.6	Inference
2.7	Rank ordering
2.8	Definitions
2.9	Batch Least Squares Algorithm
2.10	Recursive Least Squares Algorithm
2.11	Gradient Method
2.12	Learning From Example
2.13	Modified Learning From Example
2.14	Problems on logic and fuzzy systems

Lecture 06 Topics

2.11	Gradient Method
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2.11.1 Gradient Method

The **Gradient Method** is an iterative optimization method used to minimize an error function.

It updates parameters step by step in the direction that reduces error.

Error Function For a model output $\hat{y}$ and desired output y:

$$E = \frac{1}{2}(y - \hat{y})^2$$

Parameter Update Rule

$$\theta_{new} = \theta_{old} - \eta \frac{\partial E}{\partial \theta}$$

where:

- η = learning rate
- $\frac{\partial E}{\partial \theta}$ = gradient of error

2.11.2 Gradient Method

For Linear Model

Let: $$\hat{y} = \theta x$$

Error: $$E = \frac{1}{2}(y - \theta x)^2$$

Gradient: $$\frac{\partial E}{\partial \theta} = -(y - \theta x)x$$

Update:

$$\theta_{new} = \theta_{old} + \eta(y - \theta x)x$$

Worked Example

Given: x=2, y=4, θ=0 , η=0.1

Prediction: $$\hat{y} = \theta x = 0(2) = 0$$

Error term: $$y - \hat{y} = 4 - 0 = 4$$

Update: $$\theta_{new} = 0 + 0.1(4)(2)$$

$$\theta_{new} = 0.8$$

After one iteration: θ=0.8

2.11.3 Gradient Method: Python code

```
theta = 0.0
eta = 0.1


x_data = [1, 2, 3]
y_data = [2, 4, 6]


for epoch in range(20):
    for x, y in zip(x_data, y_data):
        y_pred = theta * x
        error = y - y_pred
        theta = theta + eta * error * x


print("Estimated theta:", theta)
```

2.12.1 Learning From Example

Learning from Example means adjusting fuzzy system parameters using sample input-output pairs.

Instead of manually designing the entire fuzzy system, the system learns from data.

Example Data

Temperature	Desired Fan Speed
20	Low
30	Medium
40	High

The system learns rules such as:

IF temperature is low THEN fan speed is low.

IF temperature is medium THEN fan speed is medium.

IF temperature is high THEN fan speed is high.

2.12.2 Learning From Example

General Procedure

1. Collect input-output examples.
2. Define initial membership functions.
3. Generate fuzzy rules.
4. Compare system output with desired output.
5. Adjust membership functions or rule weights.
6. Repeat until error becomes small.

Error Formula

$$E = \frac{1}{2}(y_d - y)^2$$

where:

- y_d = desired output
- y = actual output

2.12.3 Learning From Example

Simple Example

Suppose desired output = 80 and actual output = 70.

$$E = \frac{1}{2}(80 - 70)^2$$

$$E = \frac{1}{2}(100) = 50$$

The fuzzy system parameters should be adjusted to reduce this error.

2.13.1 Modified Learning From Example

Modified Learning from Example is an improved learning method that adjusts fuzzy rules, membership functions, or weights more effectively.

It is used to improve accuracy and reduce learning time.

Main Idea

Basic learning from example uses data to create rules.

Modified learning from example may also:

- Remove weak rules
- Combine similar rules
- Adjust rule weights
- Tune membership function parameters
- Reduce conflict between rules

2.13.2 Modified Learning From Example

Rule Weight

A fuzzy rule may be assigned a weight.

Example:

```
IF temperature is hot
THEN fan speed is high.
```

If this rule is very reliable: w=0.9

If less reliable: w=0.5

Weighted Rule Firing Strength

$$\alpha = w \times \mu_A(x)$$

where:

- α = firing strength
- w = rule weight
- $\mu_A(x)$ = membership value

2.13.3 Modified Learning From Example

Example

Given:

$$w = 0.8, \quad \mu_{hot}(35) = 0.7$$

Then, **firing strength** is given by:

$$\alpha = w \times \mu_A(x)$$

$$\alpha = 0.8 \times 0.7 = 0.56$$

So the rule fires with strength 0.56.

Advantages

Modified learning from example:

- Improves accuracy
- Removes unnecessary rules
- Handles noisy data
- Makes fuzzy systems more compact
- Supports adaptive fuzzy control

2.14.0 Problems on logic and fuzzy systems: Important Formulas

Classical Membership

$$\mu_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$$

Fuzzy Set

$$A = \{(x, \mu_A(x)) \mid x \in X\}$$

Fuzzy AND

$$\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$$

Fuzzy OR

$$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$$

Fuzzy NOT

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$$

Centroid Defuzzification

$$z^* = \frac{\sum z_i \mu(z_i)}{\sum \mu(z_i)}$$

Batch Least Squares

$$\theta = (X^T X)^{-1} X^T Y$$

Gradient Update

$$\theta_{new} = \theta_{old} - \eta \frac{\partial E}{\partial \theta}$$

2.14.1 Problems on logic and fuzzy systems

Problem 1: Classical Logic

A temperature is considered hot if it is at least 30°C. Determine whether 28°C and 32°C are hot.

Solution

Rule:

$$Hot = \begin{cases} 1, & T \geq 30 \\ 0, & T < 30 \end{cases}$$

For 28°C: $28 < 30$

So hot = 0.

For 32°C: $32 \geq 30$

So hot = 1.

Answer:

Temperature	Hot?
28°C	No
32°C	Yes

2.14.2 Problems on logic and fuzzy systems

Problem 2: Triangular Membership Function

Given triangular fuzzy set: a=10, b=20, c=30

Find membership value for x = 15.

Solution

Since:

$$10 < 15 \leq 20$$

$$\mu(15) = \frac{15 - 10}{20 - 10}$$

$$\mu(15) = \frac{5}{10} = 0.5$$

Answer: μ(15)=0.5

2.14.3 Problems on logic and fuzzy systems

Problem 3: Fuzzy AND

Given:

$$\mu_A(x) = 0.6$$

$$\mu_B(x) = 0.8$$

Find fuzzy AND using minimum method.

Solution:

$$\mu_{A \cap B}(x) = \min(0.6, 0.8)$$

$$\mu_{A \cap B}(x) = 0.6$$

2.14.4 Problems on logic and fuzzy systems

Problem 4: Fuzzy OR

Given:

$\mu_A(x)$=0.3

$\mu_B(x)$=0.7

Find fuzzy OR using maximum method.

Solution

$\mu_{A\cup B}(x)$ = max(0.3, 0.7)

$\mu_{A\cup B}(x)$ = 0.7

Problem 5: Fuzzy NOT

Given:

$\mu_A(x)$=0.65

Find fuzzy NOT.

Solution

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$$

$$\mu_{\bar{A}}(x) = 1 - 0.65 = 0.35$$

Problem 6: Defuzzification

Given:

Output z	Membership
10	0.2
20	0.5
30	0.3

Find crisp output using centroid method.

Solution

$$z^* = \frac{\sum z_i \mu(z_i)}{\sum \mu(z_i)}$$

$$z^* = \frac{10(0.2) + 20(0.5) + 30(0.3)}{0.2 + 0.5 + 0.3}$$

$$z^* = \frac{2 + 10 + 9}{1}$$

$$z^* = 21$$

Answer: Crisp output = 21

2.14.7 Problems on logic and fuzzy systems

Problem 7: Membership value

Find the membership value for x = 24 in triangular fuzzy set

with a = 20, b = 30, c = 40.

Answer

Since:

$$20 < 24 \leq 30$$

$$\mu(24) = \frac{24 - 20}{30 - 20}$$

$$\mu(24) = \frac{4}{10} = 0.4$$

Answer: $\mu(24)=0.4$

2.14.8 Problems on logic and fuzzy systems

Problem 8: AND, OR, and NOT

Given $\mu_A(x) = 0.4$ and $\mu_B(x) = 0.9$, find fuzzy AND, OR, and NOT A.

Answer:

AND: $\mu_{A \cap B}(x) = \min(0.4, 0.9) = 0.4$

OR: $\mu_{A \cup B}(x) = \max(0.4, 0.9) = 0.9$

NOT A: $\mu_{\bar{A}}(x) = 1 - 0.4 = 0.6$

2.14.9 Problems on logic and fuzzy systems

Problem 9: Membership value

A fuzzy set "Tall" has membership values:

Height	Membership
150 cm	0.1
160 cm	0.4
170 cm	0.7
180 cm	1.0

Find whether 170 cm is fully tall or partially tall.

Answer

Membership value of 170 cm is 0.7.

Therefore, 170 cm is partially tall with degree 0.7.

2.14.10 Problems on logic and fuzzy systems

Problem 10: Centroid

Given output values:

z	Membership
40	0.3
60	0.6
80	0.9

Find centroid output.

Answer

$$z^* = \frac{40(0.3) + 60(0.6) + 80(0.9)}{0.3 + 0.6 + 0.9}$$

$$z^* = \frac{12 + 36 + 72}{1.8}$$

$$z^* = \frac{120}{1.8} = 66.67$$

Answer:

Crisp output = 66.67

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Doubts?

THANK YOU!

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