LECTURE 07

21CSE451T

FUZZY LOGIC AND ITS APPLICATIONS

Unit 3: Rule-Base Reduction Methods

3.1	Fuzzy Systems Theory and Rule Reduction
3.2	Singular Value Decomposition
3.3	Combs Method
3.4	SVD and Combs Method Examples

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21CSE451T FUZZY LOGIC AND ITS APPLICATIONS
SYLLABUS

Unit-1 - Introduction	9 Hour
The Case for Imprecision, The Utility of Fuzzy Systems, Limitations of Fuzzy Systems, Uncertainty and Information, Fuzzy Sets and Membership, Chance versus Fuzziness - Fuzzy Sets: Fuzzy Set Operations, Properties of Fuzzy Sets, Noninteractive Fuzzy Sets, Alternative Fuzzy Set Operations - Fuzzy Relations: Crisp Relations, Fuzzy Relations, Fuzzy Tolerance and Equivalence Relations, Value Assignments, Problems on fuzzy relation - Membership function – various forms –fuzzification – defuzzification to crisp sets.	
Unit-2 - Logic and Fuzzy Systems	**9 Hour**
classical logic, fuzzy logic, fuzzy systems – Development of Membership functions: membership value assignments, intuition, Inference, rank ordering – Automated Methods for Fuzzy Systems: Definitions, Batch Least Squares Algorithm, Recursive Least Squares Algorithm, Gradient Method, Learning From Example, Modified Learning From Example, Problems on logic and fuzzy systems	
Unit-3 - Rule-Base Reduction Methods	**9 Hour**
: Fuzzy Systems Theory and Rule Reduction, Singular Value Decomposition, Combs Method, SVD and Combs Method Examples, problems on SVD and Combs method for rapid inference - Decision Making with Fuzzy Information: Fuzzy Synthetic Evaluation, Fuzzy Ordering, Nontransitive Ranking, Preference and Consensus, Multiobjective Decision Making, Decision Making under Fuzzy States and Fuzzy Actions, problems on decision making with fuzzy information.	
Unit-4 - Fuzzy Classification and Pattern Recognition	**9 Hour**
Classification by Equivalence Relations, Cluster Analysis, Cluster Validity, c-Means Clustering, Fuzzy c-Means, Classification Metric, Similarity Relations from Clustering - Pattern Recognition: Feature Analysis, Partitions of the Feature Space, Single-Sample Identification, Multifeature Pattern Recognition, problems on fuzzy classification and pattern recognition, Case Study: Hand written character recognition using fuzzy logic.	
Unit-5 - Fuzzy Control Systems	**9 Hour**
Control System Design Problem, Control (Decision) Surface, Assumptions in a Fuzzy Control System Design, Simple Fuzzy Logic Controllers, Examples of Fuzzy Control System Design, Aircraft Landing Control Problem - Fuzzy Optimization - Fuzzy Linear Regression – problems on fuzzy optimization and regression, Case study: Robot Navigation using fuzzy logic.	

Unit 3: Rule-Base Reduction Methods

3.1	Fuzzy Systems Theory and Rule Reduction
3.2	Singular Value Decomposition
3.3	Combs Method
3.4	SVD and Combs Method Examples
3.5	Problems on SVD and Combs method for rapid inference
3.6	Fuzzy Synthetic Evaluation
3.7	Fuzzy Ordering
3.8	Nontransitive Ranking
3.9	Preference and Consensus
3.10	Multiobjective Decision Making
3.11	Decision Making under Fuzzy States and Fuzzy Actions
3.12	Problems on decision making with fuzzy information

Lecture 07 Topics

3.1	Fuzzy Systems Theory and Rule Reduction
3.2	Singular Value Decomposition
3.3	Combs Method
3.4	SVD and Combs Method Examples

3.0.1 Fuzzy Systems Theory and Rule Reduction

What is Rule-Base Reduction?

A fuzzy inference system (FIS) consists of:

- Input variables
- Fuzzy membership functions
- Rule base
- Inference engine
- Defuzzification

The **rule base** is the heart of a fuzzy system. As the number of input variables increases, the number of fuzzy rules increases exponentially. This phenomenon is known as the **Curse of Dimensionality** or **Rule Explosion Problem**.

3.0.2 Fuzzy Systems Theory and Rule Reduction

Suppose a fuzzy controller has:

- 2 inputs
- Each input has 3 linguistic values

Rules required: $\mathbf{3^2 = 9}$

If there are

- 5 inputs
- Each input has 4 linguistic values

Rules required

$$\mathbf{4^5 = 1024}$$

Thus, large fuzzy systems become difficult to design, maintain and execute.

3.0.3 Fuzzy Systems Theory and Rule Reduction

Need for Rule Reduction

Rule reduction aims to

- Reduce memory usage
- Reduce computational complexity
- Increase execution speed
- Remove redundant rules
- Improve interpretability
- Maintain acceptable accuracy

3.1.1 Fuzzy Systems Theory and Rule Reduction

Fuzzy System

A fuzzy system maps inputs to outputs using IF–THEN rules.

General Rule

IF Temperature is High
AND Humidity is Medium
THEN
Fan Speed is Fast

3.1.2 Fuzzy Systems Theory and Rule Reduction

Components of Fuzzy System

1. Fuzzification

Converts crisp input into fuzzy values.

Example

Temperature = 35°C

↓

Membership

Hot = 0.8

Warm = 0.2

2. Rule Base

Contains expert knowledge.

Example

IF Temperature is High

THEN Fan Speed is High

3. Inference Engine

Evaluates all activated rules.

4. Defuzzification

Produces crisp output.

Example

Fan Speed = 850 RPM

Rule Explosion

Suppose

Temperature

- Low
- Medium
- High

Humidity

- Low
- Medium
- High

Pressure

- Low
- Medium
- High

Each variable has 3 fuzzy sets.

Total Rules: $3 \times 3 \times 3 = 27$

If another variable is added

Wind Speed

Again 3 fuzzy sets

Rules become: $\mathbf{3^4 = 81}$

Adding variables rapidly increases the number of rules.

3.1.3 Fuzzy Systems Theory and Rule Reduction

Why Rule Reduction?

Without reduction

- ✓ Large memory
- ✓ Slow inference
- ✓ Difficult optimization
- ✓ Complex hardware implementation

With reduction

- ✓ Faster computation
- ✓ Less storage
- ✓ Easier tuning
- ✓ Better understanding

3.1.4 Fuzzy Systems Theory and Rule Reduction

Methods of Rule Reduction

Several techniques exist.

1. Singular Value Decomposition (SVD)
2. Combs Method
3. Rule Merging
4. Clustering
5. Genetic Algorithms
6. Neural Learning

This lecture mainly discusses

- SVD
- Combs Method

3.1.5 Fuzzy Systems Theory and Rule Reduction

Redundant Rules

Sometimes different rules produce nearly identical outputs.

Example

Rule 1

IF Temperature is High
AND Humidity is Medium
THEN Fan Speed = Fast

Rule 2

IF Temperature is Very High
AND Humidity is Medium
THEN Fan Speed = Fast

Both rules are almost identical.
One may be removed.

Conflicting Rules

Example

Rule A

IF Temperature is High
THEN Fan Speed = Fast

Rule B

IF Temperature is High
THEN Fan Speed = Medium

These rules conflict.

Reduction methods help eliminate inconsistencies.

3.2.1 Singular Value Decomposition (SVD)

Introduction

SVD is a mathematical matrix factorization technique.

It is widely used in

- Data compression
- Machine learning
- Image processing
- Pattern recognition
- Fuzzy rule reduction

Basic Idea

A large rule matrix contains redundant information.

SVD identifies:

- Important information
- Less important information

Only important information is retained.

3.2.2 Singular Value Decomposition (SVD)

Matrix Representation

Suppose rule matrix

$$A = \begin{bmatrix} 2 & 0 \\ 1 & 1 \\ 0 & 2 \end{bmatrix}$$

SVD decomposes it as

$$A = U\Sigma V^T$$

Where

U

Left singular vectors

Represent input relationships.

Σ

Diagonal matrix

Contains singular values.

Example

$$\Sigma = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$$

Larger values represent more important information.

V

Right singular vectors

Represent output relationships.

• V^T: Transpose of an orthogonal matrix containing the right singular vectors.

3.2.3 Singular Value Decomposition (SVD)

What is SVD? Singular Value Decomposition is a powerful mathematical tool used in linear algebra for data reduction and matrix factorization. It decomposes a given matrix A into three distinct matrices:

$$A = U\Sigma V^T$$

Where:

- **A**: The original matrix (in fuzzy logic, this represents the rule base or fuzzy relation matrix).
- U: Orthogonal matrix containing the left singular vectors.
- **Σ**: Diagonal matrix containing the singular values (usually sorted from largest to smallest).
- V^T: Transpose of an orthogonal matrix containing the right singular vectors.

3.2.4 Singular Value Decomposition (SVD)

Working Principle

Step 1

Construct rule matrix

↓

Step 2

Apply SVD

↓

Step 3

Find significant singular values

↓

Step 4

Discard small singular values

↓

Step 5

Reconstruct simplified rule matrix

3.2.5 Singular Value Decomposition (SVD)

Rule Reduction

Suppose

Singular values

- 10
- 6
- 0.8
- 0.1

Threshold = 1

Keep

- 10
- 6

Discard

- 0.8
- 0.1

The resulting matrix becomes much smaller while preserving most information.

3.2.6 Singular Value Decomposition (SVD)

Advantages of SVD

- Removes redundancy
- Excellent compression
- High accuracy
- Robust
- Widely applicable

Limitations of SVD

- Computationally expensive
- Requires matrix operations
- Difficult for very large datasets
- Threshold selection is important

3.2.7 Singular Value Decomposition (SVD)

Applications of SVD

- Medical diagnosis
- Robot control
- Image compression
- Intelligent transportation
- Process control
- Decision support systems

3.2.8 Singular Value Decomposition (SVD)

SVD in Fuzzy Rule Reduction In a fuzzy system with two inputs, the rule base can be represented as a 2D matrix where the rows represent the fuzzy sets of Input 1, the columns represent the fuzzy sets of Input 2, and the matrix entries represent the output consequents.

By applying SVD to this rule matrix, we can identify the "singular values" in Σ.

- **Large singular values** represent the most important, dominant rules that define the system's behavior.
- **Small or zero singular values** represent weak, redundant, or insignificant rules.

By truncating the matrices (throwing away the smallest singular values and their corresponding vectors), we can reconstruct an *approximated* rule matrix that requires far fewer rules to compute, effectively reducing the system's complexity.

3.3.1 Combs Method

What is the Combs Method? Introduced by William E. Combs in 1997, the Combs method is a structural approach to eliminating combinatorial rule explosion. Instead of using multi-antecedent rules connected by logical ANDs (intersection), it uses single-antecedent rules connected by logical ORs (union).

The Logical Foundation The method relies on a fundamental tautology in classical logic, which states that an intersection of antecedents leading to a consequent is logically equivalent to the union of individual antecedents leading to that same consequent:

$$((p \wedge q) \Rightarrow r) \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow r))$$

(Read as: IF p AND q THEN r, is equivalent to: IF p THEN r, OR IF q THEN r).

The Reduction Impact By breaking down complex, multi-variable rules into single-variable rules, the Combs method changes the math of rule generation from multiplicative to additive. **Instead of S^N rules**, a Combs fuzzy system only requires:

Total Rules=$S \times N$

Going back to our earlier example of 5 inputs with 5 sets each: a traditional system requires 3,125 rules, while the Combs method requires only **25 rules** (5 ×5).

3.3.2 Combs Method

Traditional Rule Generation

Suppose:

Inputs

- Temperature
- Humidity
- Pressure

Each has

3 fuzzy sets

Total rules: $3^3 = 27$

Combs Method

Instead of creating 27 rules

Create separate rule groups.

- Temperature: 3 rules
- Humidity: 3 rules
- Pressure: 3 rules

Total rules: $3 + 3 + 3 = 9$

Huge reduction.

3.3.3 Combs Method

Formula

Traditional Method

$$R = m^n$$

where

m = fuzzy sets

n = variables

Combs Method

$$R = n \times m$$

Principle

Instead of

IF Temperature is High
AND Humidity is Medium
AND Pressure is Low
THEN Output

Combs separates them into

Rule Group 1

Temperature → Output

Rule Group 2

Humidity → Output

Rule Group 3

Pressure → Output

Outputs are later combined.

3.3.4 Combs Method

Advantages

- Very few rules
- Simple implementation
- Fast computation
- Suitable for embedded systems
- Easy maintenance

Disadvantages

- Ignores interaction among variables
- May reduce accuracy
- Not suitable for highly coupled systems

3.3.5 Combs Method

Traditional Rule Base	Combs Method
Exponential growth	Linear growth
Large memory	Small memory
Slow	Fast
Difficult maintenance	Easy maintenance
Complex	Simple

3.3.6 Combs Method

Applications

- ✓ Consumer electronics
- ✓ Industrial automation
- ✓ Air conditioners
- ✓ Washing machines
- ✓ Robotics
- ✓ Automotive control

3.4.1 SVD and Combs Method Examples

Example 1 (Rule Explosion)

Inputs

- Temperature
- Humidity
- Pressure

Each has

- 4 fuzzy sets.

Traditional rules

$$4^3 = 64$$

Combs

$$4 \times 3 = 12$$

Reduction

64 → 12

3.4.2 SVD and Combs Method Examples

Example 2 (Five Inputs)

Inputs

- Temperature
- Humidity
- Pressure
- Wind
- Light

Each has

- 3 fuzzy sets

Traditional

$$3^5 = 243$$

Combs

$$3 \times 5 = 15$$

Reduction: 243 → 15

3.4.3 SVD and Combs Method Examples

Example 3 (SVD)

Suppose singular values obtained are

- 15
- 7
- 2
- 0.3

Threshold = 2

Keep

- 15
- 7
- 2

Discard

- 0.3

Compression achieved while maintaining important information.

3.4.4 SVD and Combs Method Examples

Example 4 (Rule Matrix)

Suppose

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}$$

Notice

Row 1 and Row 3 are identical.

SVD detects redundancy.

One effective dimension is removed.

Memory reduces significantly.

3.4.5 SVD and Combs Method Examples

Example 5 (Fan Speed Controller)

Inputs

- Temperature
- Humidity

Each has

- 3 fuzzy sets

Traditional

$$3^2 = 9$$

Combs

$$\mathbf{3 + 3 = 6}$$

Reduction

9 → 6

3.4.6 SVD vs Combs Methods

Feature	SVD	Combs Method
Approach	Matrix decomposition	Rule decomposition
Principle	Remove redundant information	Avoid generating unnecessary combinations
Mathematics	Linear algebra	Rule grouping
Complexity	High	Low
Accuracy	High	Moderate
Computation	Heavy	Light
Memory Saving	Excellent	Good
Hardware Suitability	Moderate	Excellent
Real-Time Applications	Moderate	Excellent
Large Data Handling	Excellent	Moderate
Implementation	Difficult	Easy

Real-World Applications

1. Air Conditioner Control

- Reduces rules for temperature and humidity control.
- Enables faster response with limited hardware.

2. Washing Machines

- Simplifies fuzzy programs for load size, dirt level, and water level.
- Saves memory in embedded controllers.

3. Autonomous Vehicles

- Compresses large decision rule sets.
- Improves real-time obstacle avoidance.

4. Medical Diagnosis

- Removes redundant diagnostic rules while preserving essential knowledge.
- Speeds up inference in expert systems.

5. Industrial Process Control

- Simplifies monitoring of pressure, flow, temperature, and level.
- Reduces computational load in PLCs and embedded devices.

Summary

- **Rule explosion** occurs because the number of fuzzy rules increases exponentially with the number of input variables.
- **Rule-base reduction** improves efficiency, reduces memory requirements, and simplifies maintenance.
- **SVD** reduces the rule base by identifying and removing redundant information using matrix decomposition.
- **Combs Method** reduces rule growth from exponential to linear by decomposing multidimensional rules into independent rule groups.
- **Traditional rule count:** m^n
- **Combs Method rule count:** $n \times m$
- **SVD** generally provides higher accuracy but involves greater computational complexity.
- **Combs Method** is computationally simpler and well-suited for real-time and embedded applications.

Questions

Part A (2 Marks)

1. What is rule-base reduction in fuzzy systems?
2. Define the rule explosion problem.
3. What is Singular Value Decomposition (SVD)?
4. State the mathematical expression for SVD.
5. What are singular values?
6. Why is SVD used in fuzzy systems?
7. What is the Combs Method?
8. Write the formula for rule generation using the Combs Method.
9. Mention two advantages of rule-base reduction.
10. Compare exponential and linear rule growth.

Part B (8 Marks)

1. Explain the need for rule-base reduction in fuzzy inference systems.
2. Describe the working principle of Singular Value Decomposition (SVD) with a suitable example.
3. Explain the Combs Method for fuzzy rule-base reduction with an illustrative example.
4. Compare SVD and the Combs Method with suitable diagrams and applications.
5. Discuss the rule explosion problem and explain techniques to reduce the size of a fuzzy rule base.
6. Solve numerical problems to calculate the number of fuzzy rules using the traditional approach and the Combs Method.

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Doubts?

THANK YOU!