LECTURE 08

21CSE451T

FUZZY LOGIC AND ITS APPLICATIONS

Unit 3: Rule-Base Reduction Methods

3.5	Problems on SVD and Combs method for rapid inference
3.6	Fuzzy Synthetic Evaluation
3.7	Fuzzy Ordering
3.8	Nontransitive Ranking

Virupakshan K

+91-9840847872 | VIRU@KART.LA

21CSE451T FUZZY LOGIC AND ITS APPLICATIONS SYLLABUS

Unit-1 - Introduction	**9 Hour**
The Case for Imprecision, The Utility of Fuzzy Systems, Limitations of Fuzzy Systems, Uncertainty and Information, Fuzzy Sets and Membership, Chance versus Fuzziness - Fuzzy Sets: Fuzzy Set Operations, Properties of Fuzzy Sets, Noninteractive Fuzzy Sets, Alternative Fuzzy Set Operations - Fuzzy Relations: Crisp Relations, Fuzzy Relations, Fuzzy Tolerance and Equivalence Relations, Value Assignments, Problems on fuzzy relation - Membership function – various forms –fuzzification – defuzzification to crisp sets.	
Unit-2 - Logic and Fuzzy Systems	**9 Hour**
classical logic, fuzzy logic, fuzzy systems – Development of Membership functions: membership value assignments, intuition, Inference, rank ordering – Automated Methods for Fuzzy Systems: Definitions, Batch Least Squares Algorithm, Recursive Least Squares Algorithm, Gradient Method, Learning From Example, Modified Learning From Example, Problems on logic and fuzzy systems	
Unit-3 - Rule-Base Reduction Methods	**9 Hour**
: Fuzzy Systems Theory and Rule Reduction, Singular Value Decomposition, Combs Method, SVD and Combs Method Examples, problems on SVD and Combs method for rapid inference - Decision Making with Fuzzy Information: Fuzzy Synthetic Evaluation, Fuzzy Ordering, Nontransitive Ranking, Preference and Consensus, Multiobjective Decision Making, Decision Making under Fuzzy States and Fuzzy Actions, problems on decision making with fuzzy information.	
Unit-4 - Fuzzy Classification and Pattern Recognition	**9 Hour**
Classification by Equivalence Relations, Cluster Analysis, Cluster Validity, c-Means Clustering, Fuzzy c-Means, Classification Metric, Similarity Relations from Clustering - Pattern Recognition: Feature Analysis, Partitions of the Feature Space, Single-Sample Identification, Multifeature Pattern Recognition, problems on fuzzy classification and pattern recognition, Case Study: Hand written character recognition using fuzzy logic.	
Unit-5 - Fuzzy Control Systems	**9 Hour**
Control System Design Problem, Control (Decision) Surface, Assumptions in a Fuzzy Control System Design, Simple Fuzzy Logic Controllers, Examples of Fuzzy Control System Design, Aircraft Landing Control Problem - Fuzzy Optimization - Fuzzy Linear Regression – problems on fuzzy optimization and regression, Case study: Robot Navigation using fuzzy logic.	

Unit 3: Rule-Base Reduction Methods

3.1	Fuzzy Systems Theory and Rule Reduction
3.2	Singular Value Decomposition
3.3	Combs Method
3.4	SVD and Combs Method Examples
3.5	Problems on SVD and Combs method for rapid inference
3.6	Fuzzy Synthetic Evaluation
3.7	Fuzzy Ordering
3.8	Nontransitive Ranking
3.9	Preference and Consensus
3.10	Multiobjective Decision Making
3.11	Decision Making under Fuzzy States and Fuzzy Actions
3.12	Problems on decision making with fuzzy information

Lecture 08 Topics

3.5	Problems on SVD and Combs method for rapid inference
3.6	Fuzzy Synthetic Evaluation
3.7	Fuzzy Ordering
3.8	Nontransitive Ranking

3.5.1 Rule Count Using Combs Method

A fuzzy controller has 4 inputs. Each input has 3 fuzzy sets.

Find the number of rules using:

1. Conventional method
2. Combs method

Solution

Conventional method:

$3^4=81$

Combs method:

$3\times4=12$

Rules reduced:

$81-12=69$

3.5.2 Percentage Rule Reduction

A fuzzy system has 5 inputs and each input has 3 fuzzy sets. Find the percentage reduction using Combs method.

Solution

Conventional rules:

$$3^5=243$$

Combs rules:

$$3\times5=15$$

Reduction:

$$243-15=228$$

Percentage reduction:

$$(228/243)\times100=93.83\%$$

3.5.3 Singular Values

Given:

$$A = \begin{bmatrix} 3 & 0 \\ 4 & 0 \\ 0 & 5 \end{bmatrix}$$

Find the singular values.

Solution

First compute:

$$A^T = \begin{bmatrix} 3 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 3 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 4 & 0 \\ 0 & 5 \end{bmatrix}$$

Eigenvalues are: λ_1=25, λ_2=25

$$A^T A = \begin{bmatrix} 25 & 0 \\ 0 & 25 \end{bmatrix}$$

Singular values: σ_1=5,σ_2=5

Both components are equally important.

3.5.4 Rapid Inference Comparison

A fuzzy system has 7 inputs, each with 4 fuzzy sets.

Find:

1. Full rule count
2. Combs rule count
3. Reduction percentage

Solution

Full rule count: $4^7 = 16384$

Combs rule count: $4 \times 7 = 28$

Reduction: $16384 - 28 = 16356$

Percentage reduction: $16356 / 16384 \times 100 = 99.83\%$

This gives very fast inference because only 28 rules are evaluated instead of 16,384.

3.6.1 Fuzzy Synthetic Evaluation

1. Meaning

Fuzzy Synthetic Evaluation, FSE, is a fuzzy decision-making method used to evaluate an object or alternative based on multiple factors.

It is useful when evaluation involves vague linguistic terms such as:

- Excellent
- Good
- Average
- Poor
- Very poor

3.6.2 Fuzzy Synthetic Evaluation

2. Components of Fuzzy Synthetic Evaluation

FSE usually contains:

1. **Factor set**
2. **Evaluation set**
3. **Weight vector**
4. **Fuzzy relation matrix**
5. **Synthetic evaluation result**

3.6.3 Fuzzy Synthetic Evaluation

3. Factor Set

The factor set represents the criteria used for evaluation.

Example:

For evaluating a teacher:

$U=\{u1,u2,u3,u4\}$

where:

- $u1$ = Subject knowledge
- $u2$ = Communication
- $u3$ = Punctuality
- $u4$ = Student engagement

3.6.4 Fuzzy Synthetic Evaluation

4. Evaluation Set

The evaluation set contains rating levels.

$V=\{Excellent, Good, Average, Poor\}$

5. Weight Vector

The weight vector shows the importance of each factor.

Example:

W=**[0.4, 0.3, 0.2, 0.1]**

This means:

- Subject knowledge: 40%
- Communication: 30%
- Punctuality: 20%
- Engagement: 10%

3.6.5 Fuzzy Synthetic Evaluation

6. Fuzzy Relation Matrix

Suppose the fuzzy relation matrix is:

$$R = \begin{bmatrix} 0.5 & 0.3 & 0.2 & 0.0 \\ 0.4 & 0.4 & 0.2 & 0.0 \\ 0.3 & 0.5 & 0.2 & 0.0 \\ 0.2 & 0.4 & 0.3 & 0.1 \end{bmatrix}$$

Each row corresponds to one factor.

Each column corresponds to one evaluation grade.

3.6.6 Fuzzy Synthetic Evaluation

7. Synthetic Evaluation

The final fuzzy evaluation vector is:

$B = W \circ R$

Usually, matrix multiplication is used:

B = WR

Now:

$$B = [0.4,\ 0.3,\ 0.2,\ 0.1] \begin{bmatrix} 0.5 & 0.3 & 0.2 & 0.0 \\ 0.4 & 0.4 & 0.2 & 0.0 \\ 0.3 & 0.5 & 0.2 & 0.0 \\ 0.2 & 0.4 & 0.3 & 0.1 \end{bmatrix}$$

Calculate each component:

Excellent: 0.4(0.5)+0.3(0.4)+0.2(0.3)+0.1(0.2)=0.40

Good: 0.4(0.3)+0.3(0.4)+0.2(0.5)+0.1(0.4)=0.38

Average: 0.4(0.2)+0.3(0.2)+0.2(0.2)+0.1(0.3)=0.21

Poor: 0.4(0)+0.3(0)+0.2(0)+0.1(0.1)=0.01

So:

B=[0.40, 0.38, 0.21, 0.01]

The maximum value is 0.40 for **Excellent**.

Therefore, the teacher is evaluated as Excellent.

3.6.7 Fuzzy Synthetic Evaluation

Applications

- Employee appraisal
- Supplier selection
- Risk assessment
- Machine selection
- Medical diagnosis
- Product quality evaluation

Advantages

- Handles uncertainty
- Simple computation
- Flexible
- Multiple criteria
- Suitable for human judgments

Limitations

- Membership functions affect results
- Weight assignment is subjective

3.7.1 Fuzzy Ordering

1. Meaning

Fuzzy ordering is the process of ranking objects when their values or preferences are fuzzy rather than crisp.

In classical ordering, we can say:

$A > B$

But in fuzzy ordering, we may say:

- A is greater than B with degree 0.7
- B is greater than A with degree 0.3

3.7.2 Fuzzy Ordering

2. Fuzzy Preference Relation

A fuzzy ordering relation can be represented by a fuzzy preference matrix.

For alternatives:

$$X = \{x_1, x_2, x_3\}$$

a fuzzy preference relation may be:

$$P = \begin{bmatrix} 0.5 & 0.8 & 0.6 \\ 0.2 & 0.5 & 0.7 \\ 0.4 & 0.3 & 0.5 \end{bmatrix}$$

Here:

p_{ij} shows the degree to which x_i is preferred over x_j .

Diagonal values are usually 0.5, meaning equal preference.

3.7.3 Fuzzy Ordering

3. Interpretation

From the matrix:

- $p_{12} = 0.8$ means x_1 is preferred over x_2 with degree 0.8.
- $p_{21} = 0.2$ means x_2 is preferred over x_1 with degree 0.2.
- $p_{23} = 0.7$ means x_2 is preferred over x_3 with degree 0.7.

3.7.4 Fuzzy Ordering

4. Ranking Using Row Sum

A simple ranking score can be calculated by summing each row.

For the matrix:

$$P = \begin{bmatrix} 0.5 & 0.8 & 0.6 \\ 0.2 & 0.5 & 0.7 \\ 0.4 & 0.3 & 0.5 \end{bmatrix}$$

Scores:

$$S_1 = 0.5 + 0.8 + 0.6 = 1.9$$

$$S_2 = 0.2 + 0.5 + 0.7 = 1.4$$

$$S_3 = 0.4 + 0.3 + 0.5 = 1.2$$

Ranking:

$$x_1 > x_2 > x_3$$

3.8.1 Nontransitive Ranking

1. Meaning

In classical logic, if:

- A is preferred to B
- B is preferred to C

then usually:

- A is preferred to C

This is called **transitivity**.

But in fuzzy decision-making, preferences may become **nontransitive**.

That means:

- A may be preferred to B
- B may be preferred to C
- C may be preferred to A

This creates a preference cycle.

Rock-Paper-Scissors Example

Rock beats Scissors
Scissors beats Paper
Paper beats Rock

Ranking becomes cyclic.

3.8.2 Nontransitive Ranking

2. Example

Suppose there are three alternatives:

- A = Car
- B = Bike
- C = Bus

A person may prefer:

- Car over Bike because it is comfortable
- Bike over Bus because it is fast in traffic
- Bus over Car because it is cheaper

So: $A > B, B > C, C > A$

This is nontransitive.

3.8.3 Nontransitive Ranking

3. Fuzzy Preference Matrix Example

$$P = \begin{bmatrix} 0.5 & 0.7 & 0.4 \\ 0.3 & 0.5 & 0.8 \\ 0.6 & 0.2 & 0.5 \end{bmatrix}$$

Interpretation:

- $p_{12} = 0.7$: A preferred to B
- $p_{23} = 0.8$: B preferred to C
- $p_{31} = 0.6$: C preferred to A

So:

$A > B, B > C, C > A$

This is a nontransitive cycle.

3.8.4 Nontransitive Ranking

4. Causes of Nontransitive Ranking

Nontransitivity occurs due to:

- Multiple conflicting criteria
- Subjective preferences
- Uncertain information
- Inconsistent expert opinions
- Different weights for different comparisons

3.8.5 Nontransitive Ranking

5. Handling Nontransitive Ranking

Methods include:

1. Row-sum ranking
2. Max-min transitive closure
3. Consensus ranking
4. Dominance degree
5. Pairwise comparison adjustment
6. Defuzzification of fuzzy scores

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Doubts?

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