LECTURE 09

21CSE451T

FUZZY LOGIC AND ITS APPLICATIONS

Unit 3: Rule-Base Reduction Methods

3.9	Preference and Consensus
3.10	Multiobjective Decision Making
3.11	Decision Making under Fuzzy States and Fuzzy Actions
3.12	Problems on decision making with fuzzy information

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21CSE451T FUZZY LOGIC AND ITS APPLICATIONS
SYLLABUS

Unit-1 - Introduction	**9 Hour**
The Case for Imprecision, The Utility of Fuzzy Systems, Limitations of Fuzzy Systems, Uncertainty and Information, Fuzzy Sets and Membership, Chance versus Fuzziness - Fuzzy Sets: Fuzzy Set Operations, Properties of Fuzzy Sets, Noninteractive Fuzzy Sets, Alternative Fuzzy Set Operations - Fuzzy Relations: Crisp Relations, Fuzzy Relations, Fuzzy Tolerance and Equivalence Relations, Value Assignments, Problems on fuzzy relation - Membership function – various forms –fuzzification – defuzzification to crisp sets.	
Unit-2 - Logic and Fuzzy Systems	**9 Hour**
classical logic, fuzzy logic, fuzzy systems – Development of Membership functions: membership value assignments, intuition, Inference, rank ordering – Automated Methods for Fuzzy Systems: Definitions, Batch Least Squares Algorithm, Recursive Least Squares Algorithm, Gradient Method, Learning From Example, Modified Learning From Example, Problems on logic and fuzzy systems	
Unit-3 - Rule-Base Reduction Methods	**9 Hour**
: Fuzzy Systems Theory and Rule Reduction, Singular Value Decomposition, Combs Method, SVD and Combs Method Examples, problems on SVD and Combs method for rapid inference - Decision Making with Fuzzy Information: Fuzzy Synthetic Evaluation, Fuzzy Ordering, Nontransitive Ranking, Preference and Consensus, Multiobjective Decision Making, Decision Making under Fuzzy States and Fuzzy Actions, problems on decision making with fuzzy information.	
Unit-4 - Fuzzy Classification and Pattern Recognition	**9 Hour**
Classification by Equivalence Relations, Cluster Analysis, Cluster Validity, c-Means Clustering, Fuzzy c-Means, Classification Metric, Similarity Relations from Clustering - Pattern Recognition: Feature Analysis, Partitions of the Feature Space, Single-Sample Identification, Multifeature Pattern Recognition, problems on fuzzy classification and pattern recognition, Case Study: Hand written character recognition using fuzzy logic.	
Unit-5 - Fuzzy Control Systems	**9 Hour**
Control System Design Problem, Control (Decision) Surface, Assumptions in a Fuzzy Control System Design, Simple Fuzzy Logic Controllers, Examples of Fuzzy Control System Design, Aircraft Landing Control Problem - Fuzzy Optimization - Fuzzy Linear Regression – problems on fuzzy optimization and regression, Case study: Robot Navigation using fuzzy logic.	

Unit 3: Rule-Base Reduction Methods

3.1	Fuzzy Systems Theory and Rule Reduction
3.2	Singular Value Decomposition
3.3	Combs Method
3.4	SVD and Combs Method Examples
3.5	Problems on SVD and Combs method for rapid inference
3.6	Fuzzy Synthetic Evaluation
3.7	Fuzzy Ordering
3.8	Nontransitive Ranking
3.9	Preference and Consensus
3.10	Multiobjective Decision Making
3.11	Decision Making under Fuzzy States and Fuzzy Actions
3.12	Problems on decision making with fuzzy information

Lecture 09 Topics

3.9	Preference and Consensus
3.10	Multiobjective Decision Making
3.11	Decision Making under Fuzzy States and Fuzzy Actions
3.12	Problems on decision making with fuzzy information

3.9.1Preference and Consensus

1. Preference

A **preference relation** expresses how much one alternative is preferred over another.

For alternatives: X={x1,x2,x3}

a preference matrix is:

$$P = \begin{bmatrix} 0.5 & 0.6 & 0.8 \\ 0.4 & 0.5 & 0.7 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}$$

Here:

x1 is preferred over x2 with degree 0.6.
x1 is preferred over x3 with degree 0.8.
x2 is preferred over x3 with degree 0.7.

3.9.1b Preference and Consensus

Alternatives	Chair A	Chair B	Chair C
Chair A	0.5	0.7	0.4
Chair B	0.3	0.5	0.2
Chair C	0.6	0.8	0.5

3.9.1b Preference and Consensus

Laptop	Processing Speed	Battery Life	Portability	Price (Affordability)
L_1 (Ultra-budget)	0.5 (Average)	0.8 (Excellent)	0.9 (Excellent)	1.0 (Very Cheap)
L_2 (Workstation)	1.0 (Excellent)	0.4 (Average)	0.3 (Heavy)	0.2 (Expensive)
L_3 (All-Rounder)	0.8 (Excellent)	0.7 (Excellent)	0.7 (Good)	0.6 (Moderate)

3.9.2 Preference and Consensus

2. Consensus

Consensus means agreement among decision makers.

In group decision-making, different experts may provide different preference matrices.

The goal is to find a collective preference relation.

3.9.3 Preference and Consensus

Suppose three experts rank three alternatives.

Expert 1 preference matrix:

$$P_1 = \begin{bmatrix} 0.5 & 0.7 & 0.8 \\ 0.3 & 0.5 & 0.6 \\ 0.2 & 0.4 & 0.5 \end{bmatrix}$$

Expert 2 preference matrix:

$$P_2 = \begin{bmatrix} 0.5 & 0.6 & 0.7 \\ 0.4 & 0.5 & 0.8 \\ 0.3 & 0.2 & 0.5 \end{bmatrix}$$

Collective preference matrix using average:

$$P = \frac{P_1 + P_2}{2}$$

$$P = \begin{bmatrix} 0.5 & 0.65 & 0.75 \\ 0.35 & 0.5 & 0.7 \\ 0.25 & 0.3 & 0.5 \end{bmatrix}$$

Row sums:

$S1$=0.5+0.65+0.75=1.90
$S2$=0.35+0.5+0.7=1.55
$S3$=0.25+0.3+0.5=1.05

Final consensus ranking: $x1>x2>x3$

3.10.1 Multiobjective Decision Making

1. Meaning

Multiobjective decision making involves selecting the best alternative when multiple objectives must be considered.

These objectives may conflict with each other.

Example:

Buying a laptop involves objectives such as:

- Low cost
- High performance
- Long battery life
- Low weight
- Good display quality

A laptop with high performance may be expensive. A cheap laptop may have low performance.

3.10.2 Multiobjective Decision Making

2. Fuzzy Multiobjective Decision Making

In fuzzy multiobjective decision making, objectives are represented using fuzzy sets.

For example:

- Cost should be **low**
- Performance should be **high**
- Weight should be **light**
- Battery life should be **long**

Each objective has a membership value between 0 and 1.

3.10.3 Multiobjective Decision Making

3. Bellman-Zadeh Approach: A common approach is the Bellman-Zadeh decision model.

If there are fuzzy goals:

$$G_1, G_2, \dots, G_n$$

and fuzzy constraints:

$$C_1, C_2, \dots, C_m$$

then the fuzzy decision set is:

$$D = G_1 \cap G_2 \cap \cdots \cap G_n \cap C_1 \cap C_2 \cap \cdots \cap C_m$$

Using min operator:

$$\mu_D(x) = \min[\mu_{G_1}(x), \mu_{G_2}(x), \dots, \mu_{C_m}(x)]$$

The best alternative is the one with maximum membership in the decision set.

3.10.4 Multiobjective Decision Making

4. Example

Suppose three laptops are evaluated using two fuzzy objectives:

- Low cost
- High performance

Laptop	Low Cost Membership	High Performance Membership
A	0.8	0.6
B	0.5	0.9
C	0.7	0.7

Using min operator:

For A: $\mu D(A)$=min(0.8,0.6)=0.6

For B: $\mu D(B)$=min(0.5,0.9)=0.5

For C: $\mu D(C)$=min(0.7,0.7)=0.7

Therefore, Laptop C is the best choice.

3.10.5 Multiobjective Decision Making

5. Weighted Fuzzy Decision

If objectives have different importance, use weights.

Example:

- Cost weight = 0.4
- Performance weight = 0.6

A simple weighted score is: $$S(x) = 0.4\mu_{cost}(x) + 0.6\mu_{performance}(x)$$

For Laptop A: $S(A)$=0.4(0.8)+0.6(0.6)=0.68
For Laptop B: $S(B)$=0.4(0.5)+0.6(0.9)=0.74
For Laptop C: $S(C)$=0.4(0.7)+0.6(0.7)=0.70

Using weighted score, Laptop B is best.

This shows that different aggregation methods may produce different decisions.

3.11.1 Decision Making under Fuzzy States and Fuzzy Actions

1. Meaning

In many real-world decision problems:

- The state of the environment is uncertain or fuzzy.
- The available actions may also be fuzzy.

This is called **decision making under fuzzy states and fuzzy actions**.

3.11.2 Decision Making under Fuzzy States and Fuzzy Actions

2. Fuzzy States

A fuzzy state describes an uncertain condition.

Example:

Weather state:

- Sunny
- Cloudy
- Rainy

But the weather may not be fully one state. It can be:

- Sunny: 0.6
- Cloudy: 0.3
- Rainy: 0.1

So the fuzzy state is: $S=\{Sunny/0.6, Cloudy/0.3, Rainy/0.1\}$

3.11.3 Decision Making under Fuzzy States and Fuzzy Actions

3. Fuzzy Actions

A fuzzy action is an action whose suitability or execution is uncertain or gradual.

Example:

Travel actions:

- Drive car
- Take bus
- Walk

Their suitability may be fuzzy:

- Drive car: 0.8
- Take bus: 0.6
- Walk: 0.2

3.11.4 Decision Making under Fuzzy States and Fuzzy Actions

4. Decision Matrix

Suppose we have **actions and states**.

Action	Sunny	Cloudy	Rainy
Drive	0.8	0.7	0.4
Bus	0.6	0.8	0.7
Walk	0.9	0.5	0.1

The entries represent satisfaction or utility values.

If the **fuzzy state vector** is: S=[0.6, 0.3, 0.1]

then the **expected fuzzy score** for each action can be calculated <u>using weighted sum</u>.

3.11.4 Decision Making under Fuzzy States and Fuzzy Actions

4. Decision Matrix

Suppose we have **actions and states.**

Action	Sunny	Cloudy	Rainy
Drive	0.8	0.7	0.4
Bus	0.6	0.8	0.7
Walk	0.9	0.5	0.1

The entries represent satisfaction or utility values.

If the **fuzzy state vector** is: S=[0.6, 0.3, 0.1]

then the **expected fuzzy score** for each action can be calculated using weighted sum.

Drive: $0.6(0.8)+0.3(0.7)+0.1(0.4)=0.73$

Bus: $0.6(0.6)+0.3(0.8)+0.1(0.7)=0.67$

Walk: $0.6(0.9)+0.3(0.5)+0.1(0.1)=0.70$

Best action: Drive, because it has the highest score.

3.11.5 Decision Making under Fuzzy States and Fuzzy Actions

5. Max-Min Decision Method

Another method is the max-min approach.

For each action, take the minimum satisfaction over states, then choose the maximum.

Using the same matrix:

Action	Sunny	Cloudy	Rainy
Drive	0.8	0.7	0.4
Bus	0.6	0.8	0.7
Walk	0.9	0.5	0.1

Drive: min(0.8,0.7,0.4) =0.4
Bus: min(0.6,0.8,0.7) =0.6
Walk: min(0.9,0.5,0.1) =0.1
Maximum:- **max(0.4,0.6,0.1)** =**0.6**
Best action: Bus.

The max-min method is more conservative because it considers the worst-case performance.

3.12.1 Problems on decision making with fuzzy information

Problem 1: Fuzzy Synthetic Evaluation

A college evaluates a course using three factors:

- Teaching quality
- Course content
- Evaluation method

Weights are: W=[0.5, 0.3, 0.2]

Evaluation levels are: V={Excellent, Good, Average}

Relation matrix:

$$R = \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.4 & 0.5 & 0.1 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

Find the final evaluation.

Solution

$B=WR$

Excellent:

0.5(0.6)+0.3(0.4)+0.2(0.3)=0.48

Good:

0.5(0.3)+0.3(0.5)+0.2(0.4)=0.38

Average:

0.5(0.1)+0.3(0.1)+0.2(0.3)=0.14

So: B=[0.48, 0.38, 0.14]

The course is evaluated as <u>Excellent</u>.

3.12.2 Problems on decision making with fuzzy information

Problem 2: Fuzzy Ordering

Given the preference matrix:

$$P = \begin{bmatrix} 0.5 & 0.7 & 0.6 \\ 0.3 & 0.5 & 0.8 \\ 0.4 & 0.2 & 0.5 \end{bmatrix}$$

Rank the alternatives using row sum.

Solution

For $x1$: $S1$ = 0.5+0.7+0.6 = **1.8**

For $x2$: $S2$ = 0.3+0.5+0.8 = **1.6**

For $x3$: $S3$ = 0.4+0.2+0.5 = **1.1**

Ranking: $\mathbf{x1 > x2 > x3}$

3.12.3 Problems on decision making with fuzzy information

Problem 3: Nontransitive Ranking

Given:

$$P = \begin{bmatrix} 0.5 & 0.8 & 0.3 \\ 0.2 & 0.5 & 0.7 \\ 0.7 & 0.3 & 0.5 \end{bmatrix}$$

Check whether preferences are nontransitive.

Solution

From the matrix:

- $p_{12} = 0.8$, so x_1 is preferred to x_2 .
- $p_{23} = 0.7$, so x_2 is preferred to x_3 .
- $p_{31} = 0.7$, so x_3 is preferred to x_1 .

$$x_1 > x_2, \quad x_2 > x_3, \quad x_3 > x_1$$

This is a cycle, so the ranking is **nontransitive**.

3.12.4 Problems on decision making with fuzzy information

Problem 4: Multiobjective Decision Making

Three candidates are evaluated based on two fuzzy goals:

- High experience
- Low salary expectation

Candidate	High Experience	Low Salary
A	0.8	0.5
B	0.6	0.7
C	0.7	0.8

Using the Bellman-Zadeh min operator, select the best candidate.

SOLUTION:

Candidate A: $\mu_D(A) = \min(0.8, 0.5) = 0.5$

Candidate B: $\mu_D(B) = \min(0.6, 0.7) = 0.6$

Candidate C: $\mu_D(C) = \min(0.7, 0.8) = 0.7$

Best candidate: **C.**

3.12.5 Problems on decision making with fuzzy information

Problem 5: Decision under Fuzzy States

A farmer has three actions:

- Plant crop A
- Plant crop B
- Plant crop C

Weather states are:

- Good
- Moderate
- Poor

The fuzzy state vector is:

S=[0.5, 0.3, 0.2]

The payoff matrix is:

Action	Good	Moderate	Poor
Crop A	0.9	0.6	0.3
Crop B	0.7	0.8	0.5
Crop C	0.6	0.7	0.8

Find the best action using weighted sum.

3.12.6 Problems on decision making with fuzzy information

Solution

Crop A:

0.5(0.9)+0.3(0.6)+0.2(0.3)=0.69

Action	Good	Moderate	Poor
Crop A	0.9	0.6	0.3
Crop B	0.7	0.8	0.5
Crop C	0.6	0.7	0.8

Crop B:

0.5(0.7)+0.3(0.8)+0.2(0.5)=0.69

Crop C:

0.5(0.6)+0.3(0.7)+0.2(0.8)=0.67

Crop A and Crop B are tied with score 0.69.

If risk is considered, Crop B may be preferred because it performs better under poor weather than Crop A.

3.A Summary

Topic	Main Idea	Key Use
Fuzzy Systems Theory	Uses fuzzy IF–THEN rules	Approximate reasoning
Rule Reduction	Reduces redundant fuzzy rules	Faster inference
SVD	Matrix decomposition	Rule compression
Combs Method	Converts exponential rules into linear rules	Avoids rule explosion
Fuzzy Synthetic Evaluation	Multi-factor evaluation	Grading and assessment
Fuzzy Ordering	Ranking with fuzzy preference degrees	Decision ranking
Nontransitive Ranking	Cyclic preferences	Handling inconsistent choices
Preference and Consensus	Group agreement	Collective decision-making
Multiobjective Decision Making	Multiple fuzzy goals	Optimization under conflict
Fuzzy States and Actions	Decision under fuzzy uncertainty	Risk-based action selection

3.B1 Formula

1. **Rule explosion formula:**

$$\text{Number of full rules} = m^n$$

2. **Combs method rule count:**

$$\text{Reduced rules} = m \times n$$

3. **SVD formula:**

$$A = U\Sigma V^T$$

4. **Low-rank approximation:**

$$A_k = U_k \Sigma_k V_k^T$$

3.B2 Formula

5. **Fuzzy synthetic evaluation:**

$$B = WR$$

6. **Bellman-Zadeh fuzzy decision:**

$$\mu_D(x) = \min[\mu_{G_1}(x), \mu_{G_2}(x), \dots, \mu_{C_m}(x)]$$

7. **Weighted decision score:**

$$S(x) = \sum w_i \mu_i(x)$$

3.C Definitions

Rule Reduction

Rule reduction is the process of reducing the number of fuzzy IF–THEN rules in a fuzzy rule base by removing redundant or less significant rules. It improves computational efficiency, reduces inference time, and improves interpretability while maintaining acceptable accuracy.

SVD

Singular Value Decomposition is a matrix factorization method in which a matrix is decomposed as $A = U\Sigma V^T$. In fuzzy systems, it is used to identify dominant rule patterns and remove weak or redundant rules using low-rank approximation.

Combs Method

The Combs method reduces the exponential growth of fuzzy rules by decomposing multi-input rules into single-input rules. Instead of requiring mn rules, it requires approximately $m \times n$ rules, making fuzzy inference faster.

Fuzzy Synthetic Evaluation

Fuzzy synthetic evaluation is a method for evaluating an object based on multiple fuzzy criteria. It uses a weight vector and fuzzy relation matrix to obtain a final evaluation vector.

Nontransitive Ranking

Nontransitive ranking occurs when preferences form a cycle, such as $A > B$, $B > C$, and $C > A$. It commonly occurs in fuzzy decision-making due to conflicting criteria and subjective judgments.

3.D Comparison: SVD vs Combs Method

Feature	SVD	Combs Method
Type	Matrix decomposition method	Rule decomposition method
Main purpose	Remove redundant rule information	Reduce rule explosion
Based on	Linear algebra	Rule structure simplification
Rule reduction	By retaining major singular values	By replacing combinations with single-input rules
Accuracy	Usually good if low-rank approximation is suitable	May reduce if variable interactions are important
Complexity	Requires matrix computation	Simple and easy to implement
Best suited for	Existing rule matrices	Large multi-input fuzzy systems

3.E Sample Questions

1.A fuzzy system has 6 inputs and 4 fuzzy sets per input. Find the full rule count and Combs rule count.

2.Explain why SVD is useful in rule-base reduction.

3.Given a fuzzy relation matrix and weight vector, compute fuzzy synthetic evaluation.

4.Construct an example of nontransitive ranking.

5.Compare SVD and Combs method.

6.Solve a multiobjective fuzzy decision problem using the min operator.

7.Explain decision making under fuzzy states and fuzzy actions with an example.

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Doubts?

THANK YOU!

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