LECTURE 12

21CSE451T

FUZZY LOGIC AND ITS APPLICATIONS

Unit 4: Fuzzy Classification and Pattern Recognition

4.10	Single-Sample Identification
4.11	Multifeature Pattern Recognition
4.12	Problems on fuzzy classification and pattern recognition
4.13	Case Study: Hand written character recognition using fuzzy logic

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21CSE451T FUZZY LOGIC AND ITS APPLICATIONS SYLLABUS

Unit-1 - Introduction	**9 Hour**
The Case for Imprecision, The Utility of Fuzzy Systems, Limitations of Fuzzy Systems, Uncertainty and Information, Fuzzy Sets and Membership, Chance versus Fuzziness - Fuzzy Sets: Fuzzy Set Operations, Properties of Fuzzy Sets, Noninteractive Fuzzy Sets, Alternative Fuzzy Set Operations - Fuzzy Relations: Crisp Relations, Fuzzy Relations, Fuzzy Tolerance and Equivalence Relations, Value Assignments, Problems on fuzzy relation - Membership function – various forms –fuzzification – defuzzification to crisp sets.	
Unit-2 - Logic and Fuzzy Systems	**9 Hour**
classical logic, fuzzy logic, fuzzy systems – Development of Membership functions: membership value assignments, intuition, Inference, rank ordering – Automated Methods for Fuzzy Systems: Definitions, Batch Least Squares Algorithm, Recursive Least Squares Algorithm, Gradient Method, Learning From Example, Modified Learning From Example, Problems on logic and fuzzy systems	
Unit-3 - Rule-Base Reduction Methods	**9 Hour**
: Fuzzy Systems Theory and Rule Reduction, Singular Value Decomposition, Combs Method, SVD and Combs Method Examples, problems on SVD and Combs method for rapid inference - Decision Making with Fuzzy Information: Fuzzy Synthetic Evaluation, Fuzzy Ordering, Nontransitive Ranking, Preference and Consensus, Multiobjective Decision Making, Decision Making under Fuzzy States and Fuzzy Actions, problems on decision making with fuzzy information.	
Unit-4 - Fuzzy Classification and Pattern Recognition	**9 Hour**
Classification by Equivalence Relations, Cluster Analysis, Cluster Validity, c-Means Clustering, Fuzzy c-Means, Classification Metric, Similarity Relations from Clustering - Pattern Recognition: Feature Analysis, Partitions of the Feature Space, Single-Sample Identification, Multifeature Pattern Recognition, problems on fuzzy classification and pattern recognition, Case Study: Hand written character recognition using fuzzy logic.	
Unit-5 - Fuzzy Control Systems	**9 Hour**
Control System Design Problem, Control (Decision) Surface, Assumptions in a Fuzzy Control System Design, Simple Fuzzy Logic Controllers, Examples of Fuzzy Control System Design, Aircraft Landing Control Problem - Fuzzy Optimization - Fuzzy Linear Regression – problems on fuzzy optimization and regression, Case study: Robot Navigation using fuzzy logic.	

Unit 4: Fuzzy Classification and Pattern Recognition

4.1	Classification by Equivalence Relations
4.2	Cluster Analysis
4.3	Cluster Validity
4.4	c-Means Clustering
4.5	Fuzzy c-Means
4.6	Classification Metric
4.7	Similarity Relations from Clustering
4.8	Feature Analysis
4.9	Partitions of the Feature Space
4.10	Single-Sample Identification
4.11	Multifeature Pattern Recognition
4.12	Problems on fuzzy classification and pattern recognition
4.13	Case Study: Hand written character recognition using fuzzy logic

Lecture 12 Topics

4.10	Single-Sample Identification
4.11	Multifeature Pattern Recognition
4.12	Problems on fuzzy classification and pattern recognition
4.13	Case Study: Hand written character recognition using fuzzy logic

4.10.1 Single-Sample Identification

1. Introduction

Single-sample identification means identifying the class of one unknown sample.

Example:

Given one fruit sample, decide whether it is apple, orange, or mango.

In fuzzy logic, the sample may partially match many classes.

2. General Steps

1. Measure features of the sample.
2. Convert features into fuzzy membership values.
3. Compare with known class patterns.
4. Choose class with highest similarity or membership.

4.10.2 Single-Sample Identification

3. Fuzzy Matching

Suppose a sample has feature vector x.

Class prototype is v_i.

Similarity can be calculated using distance:

$$S_i(x) = \frac{1}{1 + d(x, v_i)}$$

where:

- $S_i(x)$ = similarity to class i
- $d(x, v_i)$ = distance from sample to class prototype

The class with highest similarity is selected.

The standard **Euclidean distance** formula between an unknown sample $\mathbf{x} = [x_1, x_2]$and a prototype $\mathbf{p} = [p_1, p_2]$is:

$$d(\mathbf{x}, \mathbf{p}) = \sqrt{(x_1 - p_1)^2 + (x_2 - p_2)^2}$$

4.10.3 Single-Sample Identification

$$d(\mathbf{x}, \mathbf{p}) = \sqrt{(x_1 - p_1)^2 + (x_2 - p_2)^2}$$

$$S_i(x) = \frac{1}{1 + d(x, v_i)}$$

Worked Example

Known class prototypes:

Class	Weight	Sweetness
Apple	150	0.8
Orange	180	0.6

Unknown sample: $x = [155, 0.75]$

Distance to Apple:

$$d_A = \sqrt{(155 - 150)^2 + (0.75 - 0.8)^2}$$

$$d_A = \sqrt{25 + 0.0025} = 5.00025$$

Distance to Orange:

$$d_O = \sqrt{(155 - 180)^2 + (0.75 - 0.6)^2}$$

$$d_O = \sqrt{625 + 0.0225} = 25.00045$$

Similarity to Apple:

$$S_A = \frac{1}{1 + 5.00025} = 0.167$$

Similarity to Orange:

$$S_O = \frac{1}{1 + 25.00045} = 0.038$$

So the sample is identified as Apple.

4.10.4 Single-Sample Identification

Important Note on Scaling

Weight is much larger than sweetness. This may dominate the distance.

So features should often be normalized.

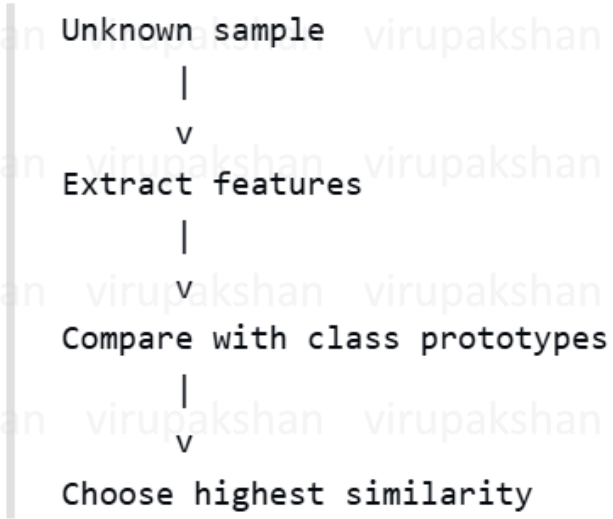
Example:

- Weight scaled between 0 and 1.
- Sweetness already between 0 and 1.

4.10.5 Single-Sample Identification

```python
import numpy as np

apple = np.array([150, 0.8])
orange = np.array([180, 0.6])
sample = np.array([155, 0.75])

def similarity(x, v):
    d = np.linalg.norm(x - v)
    return 1 / (1 + d)

s_apple = similarity(sample, apple)
s_orange = similarity(sample, orange)

print("Similarity to Apple:", s_apple)
print("Similarity to Orange:", s_orange)

if s_apple > s_orange:
    print("Identified as Apple")
else:
    print("Identified as Orange")
```

Diagram




```
import numpy as np
apple = np.array([150, 0.8])
orange = np.array([180, 0.6])
sample = np.array([155, 0.75])

def similarity(x, v):
    d = np.linalg.norm(x - v)
    return 1 / (1 + d)

s_apple = similarity(sample, apple)
s_orange = similarity(sample, orange)

print("Similarity to Apple:", s_apple)
print("Similarity to Orange:", s_orange)

if s_apple > s_orange:
    print("Identified as Apple")
else:
    print("Identified as Orange")
```

4.10.6 Single-Sample Identification

Quick Revision Points

- Single-sample identification classifies one unknown object.
- Fuzzy identification gives degrees of match.
- Similarity can be based on distance.
- Highest similarity determines the class.
- Feature scaling is important.

4.10.7 Single-Sample Identification

Questions and Answers

Q1. What is single-sample identification?

It is the process of identifying the class of one unknown sample.

Q2. Why is fuzzy matching useful?

Because a sample may partially match more than one class.

Q3. What is the role of normalization?

Normalization prevents large-valued features from dominating distance calculations.

4.11.1 Multifeature Pattern Recognition

1. Introduction

Pattern recognition means identifying patterns in data.

Multifeature pattern recognition uses many features at the same time.

Example:

To recognize a handwritten digit, we may use:

- Height
- Width
- Number of loops
- Stroke direction
- Symmetry
- Edge density

4.11.2 Multifeature Pattern Recognition

2. Pattern Vector

A pattern is represented by a vector:

$$x=[x1,x2,x3,...,xn]$$

where each xi is a feature.

Example:

For digit recognition:

x=[height, width, loop count, slant]

4.11.3 Multifeature Pattern Recognition

3. Fuzzy Rule-Based Pattern Recognition

A fuzzy rule has the form:

```
IF condition THEN class
```

Example:

```
IF loop_count is high AND height is tall THEN digit is 8
```

Another example:

```
IF loop_count is zero AND vertical_stroke is high THEN digit is 1
```

4.11.4 Multifeature Pattern Recognition

4. Fuzzy Inference

Fuzzy inference combines rules to make a decision.

Basic steps:

1. Fuzzification
2. Rule evaluation
3. Aggregation
4. Defuzzification or class selection

4.11.5 Multifeature Pattern Recognition

5. Combining Multiple Features

For AND operation, use minimum:

$$\mu A \cap B(x) = \min(\mu A(x), \mu B(x))$$

For OR operation, use maximum:

$$\mu A \cup B(x) = \max(\mu A(x), \mu B(x))$$

4.11.6 Multifeature Pattern Recognition

Worked Example

Suppose we identify **digit 8** using two fuzzy features:

- Loop count is high
- Symmetry is high

For a sample:

Feature	Membership
Loop count high	0.9
Symmetry high	0.7

Rule:

```
IF loop count is high AND symmetry is high THEN digit is 8
```

Rule firing strength: $\alpha = \min(0.9, 0.7) = 0.7$

So the sample belongs to **digit 8** with **degree 0.7.**

4.11.7 Multifeature Pattern Recognition

```
loop_high = 0.9
symmetry_high = 0.7

digit_8_strength = min(loop_high, symmetry_high)

print("Degree of digit 8:", digit_8_strength)
```

loop_high = 0.9
symmetry_high = 0.7

digit_8_strength = min(loop_high, symmetry_high)

print("Degree of digit 8:", digit_8_strength)

4.11.8 Multifeature Pattern Recognition

Larger Example

Rules:

Rule 1:

IF **loop_count** is high AND **symmetry** is high THEN digit is 8

Rule 2:

IF **loop_count** is medium AND **top_loop** is high THEN digit is 9

Rule 3:

IF **loop_count** is zero AND **vertical_stroke** is high THEN digit is 1

For a sample:

Feature	Membership
loop_count high	0.8
symmetry high	0.7
loop_count medium	0.4
top_loop high	0.5
loop_count zero	0.1
vertical_stroke high	0.9

Rule strengths:

$R1$=min(0.8,0.7)=0.7

$R2$=min(0.4,0.5)=0.4

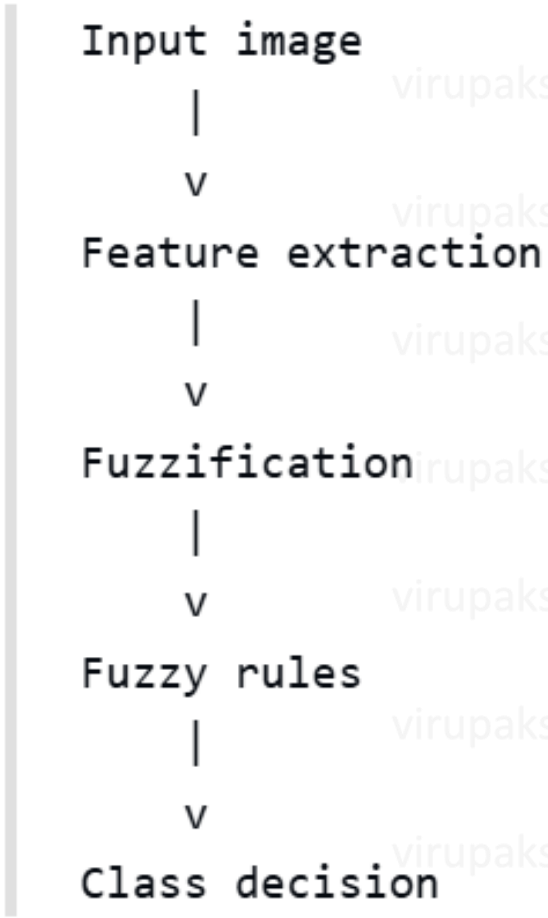
$R3$=min(0.1,0.9)=0.1

Highest rule strength is 0.7,

so class = digit 8.

4.11.9 Multifeature Pattern Recognition

Diagram



Quick Revision Points

- Pattern recognition identifies meaningful patterns.
- Multifeature recognition uses many features.
- Fuzzy rules can model uncertain patterns.
- AND is often implemented using minimum.
- OR is often implemented using maximum.
- The class with highest rule strength is selected.

4.11.10 Multifeature Pattern Recognition

Questions and Answers

Q1. What is a pattern vector?

A pattern vector is a list of feature values representing an object.

Q2. What is fuzzy rule-based classification?

It classifies objects using rules with fuzzy conditions.

Q3. How is fuzzy AND commonly calculated?

Using the minimum operator.

4.12.1 Problems on Fuzzy Classification and Pattern Recognition

Problem 1: Fuzzy Similarity Relation

Three objects have feature values:

Object	Feature Value
A	20
B	25
C	80

Use:

$$R(x_i, x_j) = 1 - \frac{|x_i - x_j|}{100}$$

Find the **similarity matrix**.

Solution:

$$R(A,A) = 1, \quad R(B,B) = 1, \quad R(C,C) = 1$$

$$R(A,B) = 1 - \frac{|20-25|}{100} = 0.95$$

$$R(A,C) = 1 - \frac{|20-80|}{100} = 0.40$$

$$R(B,C) = 1 - \frac{|25-80|}{100} = 0.45$$

Matrix:

	A	B	C
A	1.00	0.95	0.40
B	0.95	1.00	0.45
C	0.40	0.45	1.00

4.12.2 Problems on Fuzzy Classification and Pattern Recognition

Problem 2: Membership Using Triangular Function

For fuzzy set "Tall": a=160, b=180, c=200

Find membership for height 170.

Solution

Since 160 <170 ≤180:

$$\mu_{Tall}(170) = \frac{170 - 160}{180 - 160} = \frac{10}{20} = 0.5$$

So height **170** belongs to **Tall** with **degree 0.5**.

4.12.3 Problems on Fuzzy Classification and Pattern Recognition

Problem 3: c-Means Clustering

Data points: 2, 3, 9, 10

Let initial centers be: v_1=2, v_2=9

Find new clusters and updated centers.

Solution

Distances:

Point	To v_1 =2	To v_2 =9	Cluster
2	0	7	C1
3	1	6	C1
9	7	0	C2
10	8	1	C2

Clusters:

- C1 = {2, 3}
- C2 = {9, 10}

Updated centers:

$$v_1 = \frac{2+3}{2} = 2.5$$

$$v_2 = \frac{9+10}{2} = 9.5$$

4.12.4 Problems on Fuzzy Classification and Pattern Recognition

Problem 4: Fuzzy c-Means Membership

Given centers: v_1=5, v_2=15

For point: x=7

Let: m=2

Find membership in cluster 1 and cluster 2.

Solution:

Distances:

$d1$=|7−5|=2

$d2$=|7−15|=8

Membership in cluster 1:

$$u_1 = \frac{1}{1+\left(\frac{d_1}{d_2}\right)^2}$$

$$u_1 = \frac{1}{1+\left(\frac{2}{8}\right)^2} = \frac{1}{1+0.0625} = 0.941$$

Membership in cluster 2:

$$u_2 = 1 - 0.941 = 0.059$$

4.12.5 Problems on Fuzzy Classification and Pattern Recognition

Problem 5: Classification Metrics

Given:

- TP = 30
- TN = 50
- FP = 10
- FN = 10

Find accuracy, precision, recall, and F1-score.

$$Accuracy = \frac{TP + TN}{TP + TN + FP + FN}$$

$$Precision = \frac{TP}{TP + FP}$$

$$F1 = \frac{2 \times Precision \times Recall}{Precision + Recall}$$

$$Recall = \frac{TP}{TP + FN}$$

Solution:

$$Accuracy = \frac{30 + 50}{30 + 50 + 10 + 10} = \frac{80}{100} = 0.8$$

$$Precision = \frac{30}{30 + 10} = \frac{30}{40} = 0.75$$

$$Recall = \frac{30}{30 + 10} = \frac{30}{40} = 0.75$$

$$F1 = \frac{2(0.75)(0.75)}{0.75 + 0.75} = 0.75$$

4.12.Q1 Problems on Fuzzy Classification and Pattern Recognition

Q1. Define fuzzy classification.

Answer: Fuzzy classification allows an object to belong to more than one class with different membership degrees.

Q2. Explain the difference between c-means and fuzzy c-means.

Answer:

c-Means	Fuzzy c-Means
Hard clustering	Soft clustering
One point belongs to one cluster	One point can belong to many clusters
Membership is 0 or 1	Membership is between 0 and 1

4.12.Q2 Problems on Fuzzy Classification and Pattern Recognition

Q3. What is the membership condition in FCM?

Answer:

$$\sum_{i=1}^{c} u_{ik} = 1$$

Q4. What is the purpose of cluster validity?

Answer: Cluster validity evaluates whether the formed clusters are meaningful, compact, and well separated.

Q5. Why is fuzzy logic useful in pattern recognition?

Answer: Because real-world patterns are often uncertain, noisy, or overlapping. Fuzzy logic handles partial truth and gradual class boundaries.

4.13.1 Case Study: Handwritten Character Recognition Using Fuzzy Logic

1. Problem Statement

Handwritten character recognition means identifying letters or digits written by humans.

Example:

Recognize handwritten digits:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

This is difficult because handwriting differs from person to person.

For example, the digit 2 may be written in many styles.

Fuzzy logic is useful because characters may not have exact shapes.

4.13.2 Case Study: Handwritten Character Recognition Using Fuzzy Logic

2. Why Use Fuzzy Logic?

Handwritten characters are:

- Noisy
- Irregular
- Slanted
- Different in size
- Sometimes incomplete
- Similar to other characters

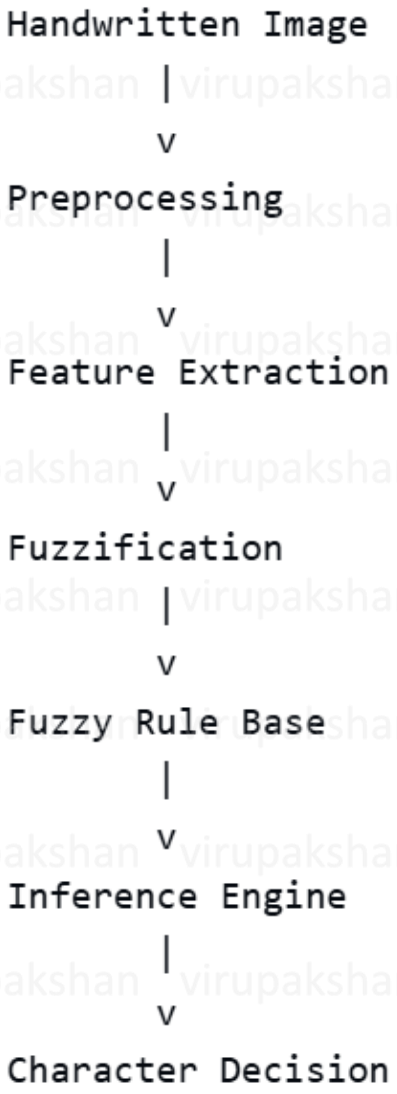
Example:

- 1 and 7 can look similar.
- 0 and 6 can look similar.
- 5 and 6 can look similar.
- 8 and B can look similar.

Fuzzy logic allows partial matching.

4.13.3 Case Study: Handwritten Character Recognition Using Fuzzy Logic

3. System Architecture



4.13.4 Case Study: Handwritten Character Recognition Using Fuzzy Logic

4. Step 1: Image Acquisition

The handwritten character is captured using:

- Scanner
- Camera
- Touchscreen
- Digital pen

Example input:

Image of handwritten digit 8

4.13.5 Case Study: Handwritten Character Recognition Using Fuzzy Logic

5. Step 2: Preprocessing

Preprocessing improves the image quality.

Common preprocessing steps:

a. Grayscale Conversion

Convert color image to grayscale.

b. Binarization

Convert image into black and white.

Pixel values:

- 1 for ink/stroke
- 0 for background

c. Noise Removal

Remove unwanted dots or marks.

d. Normalization

Resize the image to a fixed size, such as 28 × 28 pixels.

e. Thinning

Reduce thick strokes to single-pixel width.

4.13.6 Case Study: Handwritten Character Recognition Using Fuzzy Logic

6. Step 3: Feature Extraction

Important features for handwritten characters include:

Feature	Meaning
Height	Vertical size of character
Width	Horizontal size of character
Aspect ratio	Height divided by width
Number of loops	Closed regions in character
Endpoints	Stroke ending points
Junctions	Points where strokes meet
Horizontal strokes	Amount of horizontal writing
Vertical strokes	Amount of vertical writing
Curvature	Amount of curve
Symmetry	Similarity between left and right halves
Slant angle	Tilt of writing

4.13.7 Case Study: Handwritten Character Recognition Using Fuzzy Logic

7. Example Features for Digits

Digit	Typical Features
0	One loop, high roundness
1	Vertical stroke, no loop
2	Top curve, diagonal stroke, bottom horizontal stroke
3	Two right curves, no closed loop
4	Junction, vertical and diagonal strokes
5	Top horizontal stroke, curve at bottom
6	One loop at bottom, curved stroke
7	Top horizontal stroke, diagonal stroke
8	Two loops
9	One loop at top, descending stroke

4.13.8 Case Study: Handwritten Character Recognition Using Fuzzy Logic

8. Step 4: Fuzzification

Convert numerical features into fuzzy values.

Example:

Feature: loop count

Loop Count	No Loop	One Loop	Two Loops
0	1.0	0.0	0.0
1	0.0	1.0	0.0
2	0.0	0.0	1.0

Feature: vertical stroke strength

Value	Low	Medium	High
0.2	0.8	0.3	0.0
0.5	0.2	1.0	0.2
0.9	0.0	0.2	0.9

4.13.9 Case Study: Handwritten Character Recognition Using Fuzzy Logic

9. Step 5: Fuzzy Rule Base
Sample fuzzy rules:

Rule 1:

IF loop count is two AND symmetry is high

THEN character is 8

Rule 2:

IF loop count is one AND roundness is high

THEN character is 0

Rule 3:

IF loop count is zero AND vertical stroke is high

THEN character is 1

Rule 4:

IF top horizontal stroke is high AND diagonal stroke is high

THEN character is 7

Rule 5:

IF one loop is high AND lower loop position is high

THEN character is 6

Rule 6:

IF one loop is high AND upper loop position is high

THEN character is 9

4.13.10 Case Study: Handwritten Character Recognition Using Fuzzy Logic

10. Step 6: Inference

Use fuzzy AND with minimum.

Example:

For a digit image:

Feature	Membership
Two loops	0.85
Symmetry high	0.75
Vertical stroke high	0.30

Rule for digit 8:

```
IF two loops AND symmetry high THEN digit is 8
```

Rule strength:

$$\alpha_8 = \min(0.85, 0.75) = 0.75$$

Rule for digit 1:

```
IF no loop AND vertical stroke high THEN digit is 1
```

Suppose no loop membership is 0.10.

$$\alpha_1 = \min(0.10, 0.30) = 0.10$$

Since 0.75 is higher than 0.10, the digit is identified as 8.

4.13.11 Case Study: Handwritten Character Recognition Using Fuzzy Logic

11. Step 7: Decision Making

The final class is selected by maximum membership: $Class = \arg\max_i(\alpha_i)$

where α_i is the rule strength for class i.

Worked Case Study Example

A handwritten digit has the following fuzzy feature values:

Feature	Membership Value
Two loops	0.90
One loop	0.20
No loop	0.05
Symmetry high	0.80
Vertical stroke high	0.40
Upper loop high	0.25
Lower loop high	0.30

4.13.12 Case Study: Handwritten Character Recognition Using Fuzzy Logic

Rules:

```
R1: IF two loops AND symmetry high THEN digit is 8
R2: IF one loop AND upper loop high THEN digit is 9
R3: IF one loop AND lower loop high THEN digit is 6
R4: IF no loop AND vertical stroke high THEN digit is 1
```

Rule strengths:

$$R1 = \min(0.90, 0.80) = 0.80$$

$$R2 = \min(0.20, 0.25) = 0.20$$

$$R3 = \min(0.20, 0.30) = 0.20$$

$$R4 = \min(0.05, 0.40) = 0.05$$

Highest strength: R1 = 0.80

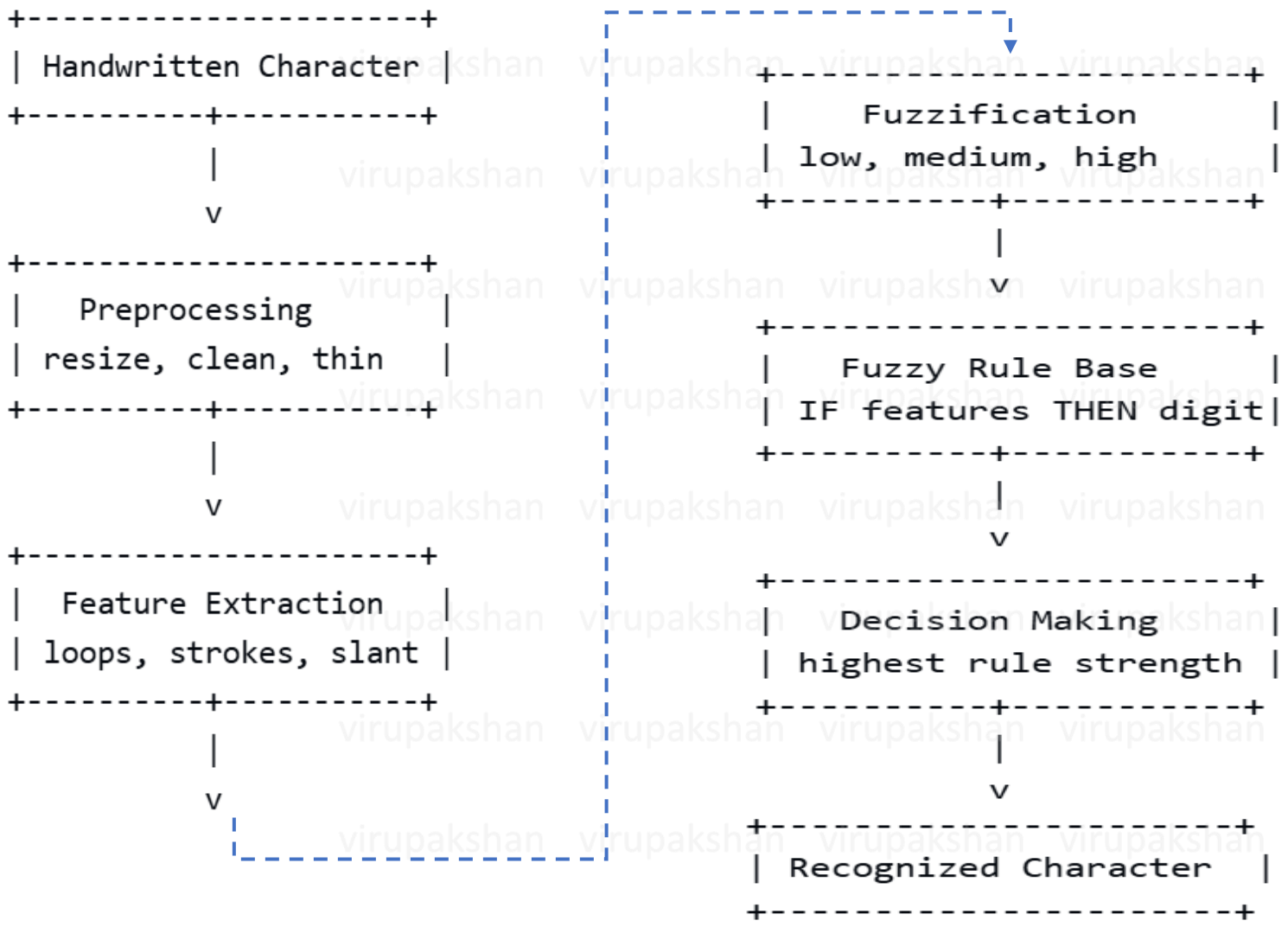
Therefore, the character is recognized as digit 8.

4.13.13 Python Code for Simple Fuzzy Digit Recognition

```
def fuzzy_and(a, b):
    return min(a, b)

# Fuzzy feature values for one handwritten sample
features = {
    "two_loops": 0.90,
    "one_loop": 0.20,
    "no_loop": 0.05,
    "symmetry_high": 0.80,
    "vertical_stroke_high": 0.40,
    "upper_loop_high": 0.25,
    "lower_loop_high": 0.30
}

# Rule strengths
digit_8 = fuzzy_and(features["two_loops"], features["symmetry_high"])
digit_9 = fuzzy_and(features["one_loop"], features["upper_loop_high"])
digit_6 = fuzzy_and(features["one_loop"], features["lower_loop_high"])
digit_1 = fuzzy_and(features["no_loop"], features["vertical_stroke_high"])
```

```
scores = {
    "8": digit_8,
    "9": digit_9,
    "6": digit_6,
    "1": digit_1
}

predicted_digit = max(scores, key=scores.get)

print("Scores:", scores)
print("Predicted Digit:", predicted_digit)
```

4.13.14 Diagram of Fuzzy Recognition Process



4.13.15 Case Study: Handwritten Character Recognition Using Fuzzy Logic

Advantages:

- Handles uncertain handwriting.
- Allows partial matching.
- Works well with rule-based systems.
- Easy to understand.
- Can combine many features.
- Useful when exact mathematical boundaries are difficult.

Limitations:

- Rule design can be difficult.
- Too many rules may be needed.
- Performance depends on good feature extraction.
- May not outperform deep learning for large image datasets.
- Membership functions must be carefully designed.

4.13.16 Case Study: Handwritten Character Recognition Using Fuzzy Logic

Quick Revision Points

- Handwritten recognition is a pattern recognition problem.
- Fuzzy logic handles uncertainty in handwriting.
- Features include loops, strokes, symmetry, and slant.
- Fuzzy rules classify the character.
- The class with highest rule strength is selected.
- Fuzzy systems are interpretable but depend on good rules.

4.13.17 Case Study: Handwritten Character Recognition Using Fuzzy Logic

Questions and Answers

Q1. Why is handwritten character recognition difficult?

Because handwriting varies in shape, size, slant, thickness, and style.

Q2. Why is fuzzy logic useful for handwritten recognition?

Because it supports partial matching and handles uncertain patterns.

Q3. Give two features used in handwritten digit recognition.

Loop count and stroke direction.

Q4. Write a fuzzy rule for recognizing digit 8.

```
IF loop count is two AND symmetry is high THEN digit is 8
```

Q5. If two loops = 0.7 and symmetry high = 0.9, what is the rule strength for digit 8?

Using fuzzy AND: α=min(0.7,0.9)=0.7

4.A Key Concepts

Topic	Main Idea
Equivalence Relations	Group objects based on similarity
Cluster Analysis	Find natural groups in data
Cluster Validity	Evaluate clustering quality
c-Means	Hard clustering using cluster centers
Fuzzy c-Means	Soft clustering using membership degrees
Classification Metrics	Measure classifier performance
Similarity Relations	Measure how similar objects are
Feature Analysis	Study useful properties of objects
Feature Space Partitions	Divide feature space into regions
Single-Sample Identification	Classify one unknown sample
Multifeature Recognition	Use many features for recognition
Case Study	Fuzzy handwritten character recognition

4.B.1 Important Formulas

Similarity Relation

$$R(x_i, x_j) = 1 - \frac{|x_i - x_j|}{100}$$

Euclidean Distance

$$d(x, y) = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$$

c-Means Objective Function

$$J = \sum_{i=1}^{c} \sum_{x_k \in C_i} ||x_k - v_i||^2$$

c-Means Cluster Center

$$v_i = \frac{1}{|C_i|} \sum_{x_k \in C_i} x_k$$

Fuzzy c-Means Objective Function

$$J_m = \sum_{k=1}^{n} \sum_{i=1}^{c} u_{ik}^m ||x_k - v_i||^2$$

Fuzzy c-Means Center

$$v_i = \frac{\sum_{k=1}^{n} u_{ik}^m x_k}{\sum_{k=1}^{n} u_{ik}^m}$$

4.B.2 Important Formulas

FCM Membership

$$u_{ik} = \frac{1}{\sum_{j=1}^{c} \left(\frac{||x_k - v_i||}{||x_k - v_j||} \right)^{\frac{2}{m-1}}}$$

Accuracy

$$Accuracy = \frac{TP + TN}{TP + TN + FP + FN}$$

Precision

$$Precision = \frac{TP}{TP + FP}$$

Recall

$$Recall = \frac{TP}{TP + FN}$$

F1-Score

$$F1 = \frac{2 \times Precision \times Recall}{Precision + Recall}$$

Fuzzy AND

$$\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$$

Fuzzy OR

$$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$$

4.C1 Q&A

1. What is fuzzy classification?

It is classification where an object can belong to multiple classes with different membership degrees.

2. What is clustering?

Clustering is grouping similar objects without predefined labels.

3. What is fuzzy c-means?

It is a clustering method where each data point has membership values in all clusters.

4. What is cluster validity?

It measures the quality of clustering results.

5. What is a feature?

A feature is a measurable property of an object.

6. What is a fuzzy partition?

A fuzzy partition divides feature space into overlapping regions with membership degrees.

7. What is pattern recognition?

Pattern recognition is the process of identifying patterns or classes in data.

4.C2 Q&A

Q1. Compare c-means and fuzzy c-means.

Point	c-Means	Fuzzy c-Means
Type	Hard clustering	Fuzzy clustering
Membership	0 or 1	Between 0 and 1
Assignment	One cluster only	Multiple clusters
Output	Cluster labels	Membership matrix
Flexibility	Less flexible	More flexible

Q2. Explain the role of features in pattern recognition.

Features describe the important properties of objects. In pattern recognition, raw data is converted into feature vectors. These feature vectors are then used to classify or recognize patterns.

Example: In handwritten digit recognition, useful features include loop count, slant, stroke direction, height, and width.

Q3. Explain why fuzzy logic is useful in real-world classification.

Real-world data often has uncertainty and overlapping class boundaries. Fuzzy logic allows partial membership, so it can model vague concepts such as tall, medium, similar, bright, and near.

4.C3 Q&A

1. Define fuzzy equivalence relation.
A fuzzy equivalence relation is a fuzzy relation that is reflexive, symmetric, and transitive.

2. What are the three properties of equivalence relation?
Reflexivity, symmetry, and transitivity.

3. What is intra-cluster similarity?
It is the similarity among objects within the same cluster.

4. What is inter-cluster separation?
It is the distance or difference between different clusters.

5. What does a silhouette score of 0 mean?
It means the point lies near the boundary between two clusters.

6. What happens when fuzziness parameter m increases?
The clustering becomes more fuzzy and membership values become more evenly distributed.

7. What is a confusion matrix?
It is a table showing correct and incorrect predictions of a classifier.

8. What is precision?
Precision is the ratio of true positives to all predicted positives.

9. What is recall?
Recall is the ratio of true positives to all actual positives.

10. What is feature extraction?
Feature extraction is the process of converting raw data into useful measurable features.

Unit 4: Fuzzy Classification and Pattern Recognition

4.1	Classification by Equivalence Relations
4.2	Cluster Analysis
4.3	Cluster Validity
4.4	c-Means Clustering
4.5	Fuzzy c-Means
4.6	Classification Metric
4.7	Similarity Relations from Clustering
4.8	Feature Analysis
4.9	Partitions of the Feature Space
4.10	Single-Sample Identification
4.11	Multifeature Pattern Recognition
4.12	Problems on fuzzy classification and pattern recognition
4.13	Case Study: Hand written character recognition using fuzzy logic

Lecture 12 Topics

4.10	Single-Sample Identification
4.11	Multifeature Pattern Recognition
4.12	Problems on fuzzy classification and pattern recognition
4.13	Case Study: Hand written character recognition using fuzzy logic

LECTURE 12

21CSE451T

FUZZY LOGIC AND ITS APPLICATIONS

Doubts?

THANK YOU!

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